USC Viterbi
School of Engineering
Center for Cyber-Physical Systems and the Internet of Things

SOFTWARE DEFINED RADIO

USR SDR WORKSHOP, SEPTEMBER 2017
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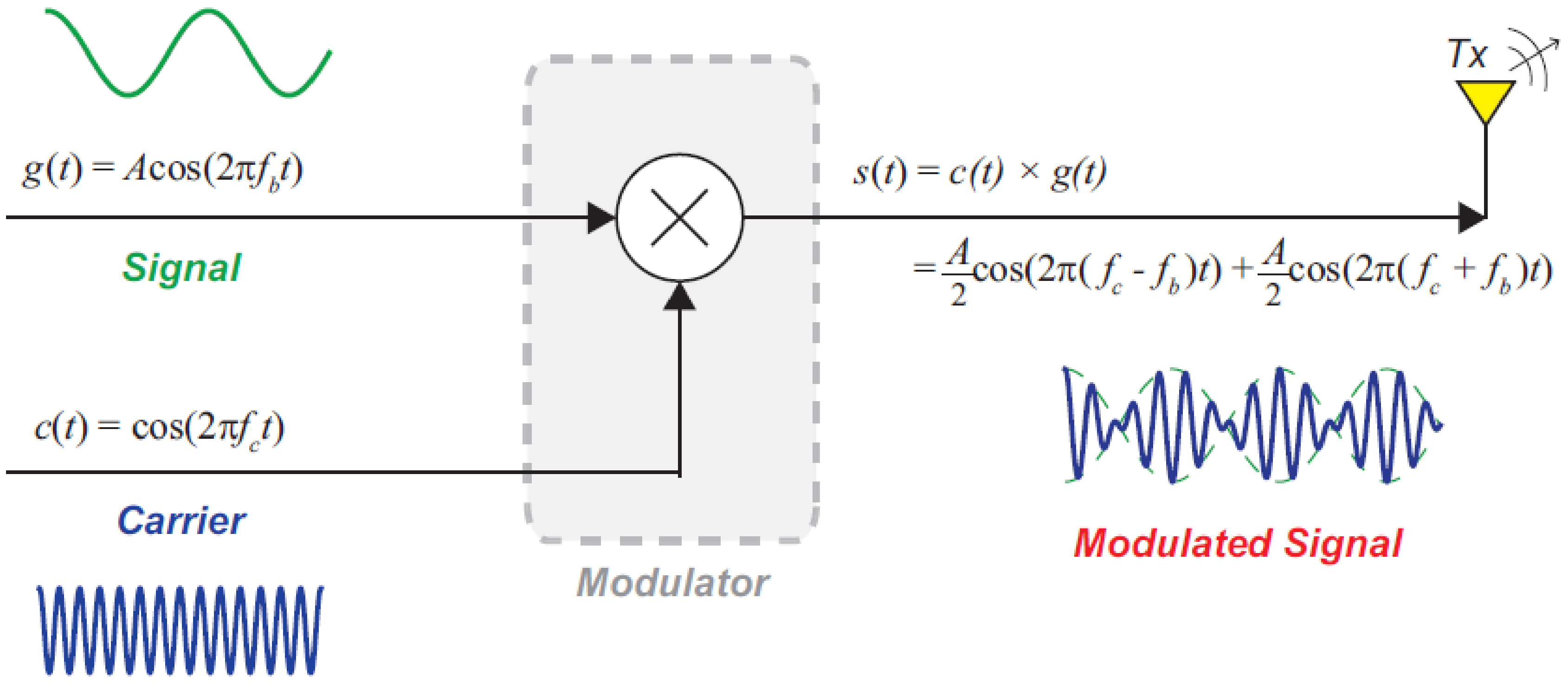
SESSION 2: AM MODULATION, REVIEW OF CONCEPT

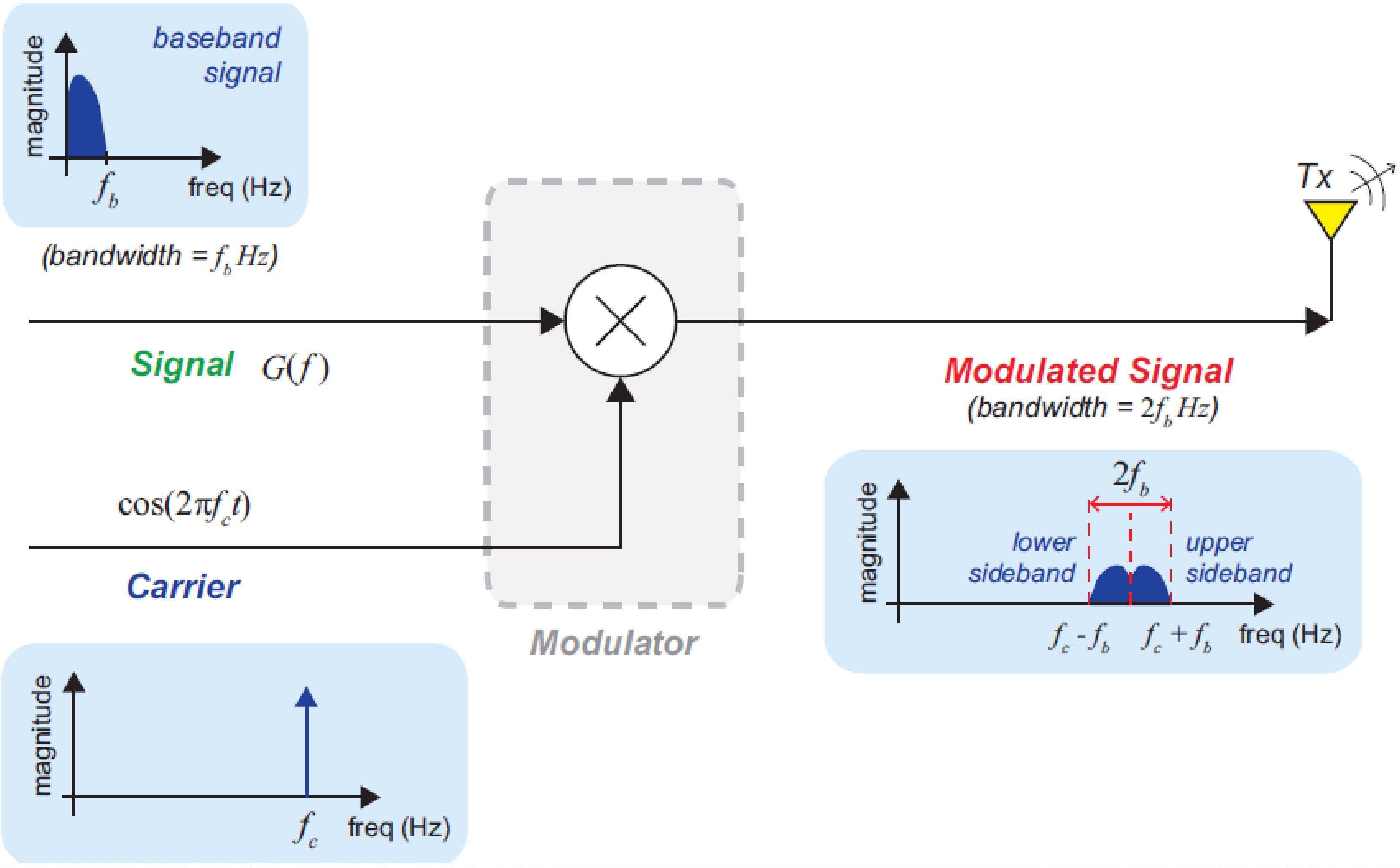
REAL IMPLEMENTATION

QUADRATURE AMPLITUDE MODULATION

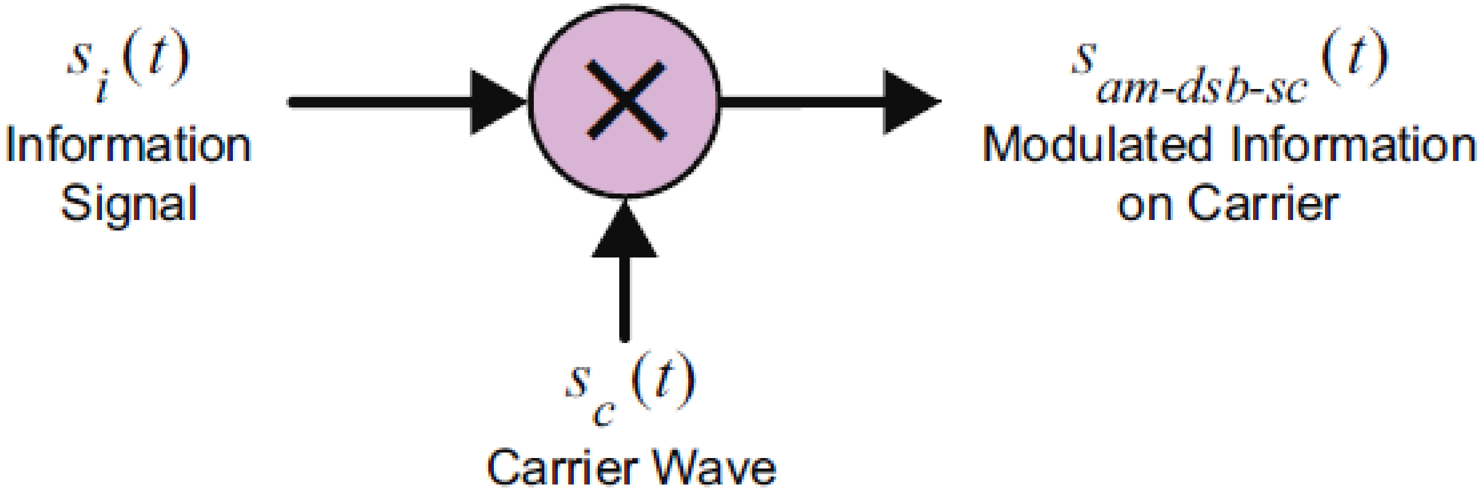
- AM is poor efficiency since it use double of bandwidth.
- But is on of the simplest modulation systems, AM-DSB-SC.



BANDWIDTH EFFICIENCY



DSB-SC



$$s_i(t) = A_i \cos(2\pi f_i t) = A_i \cos(\omega_i t)$$

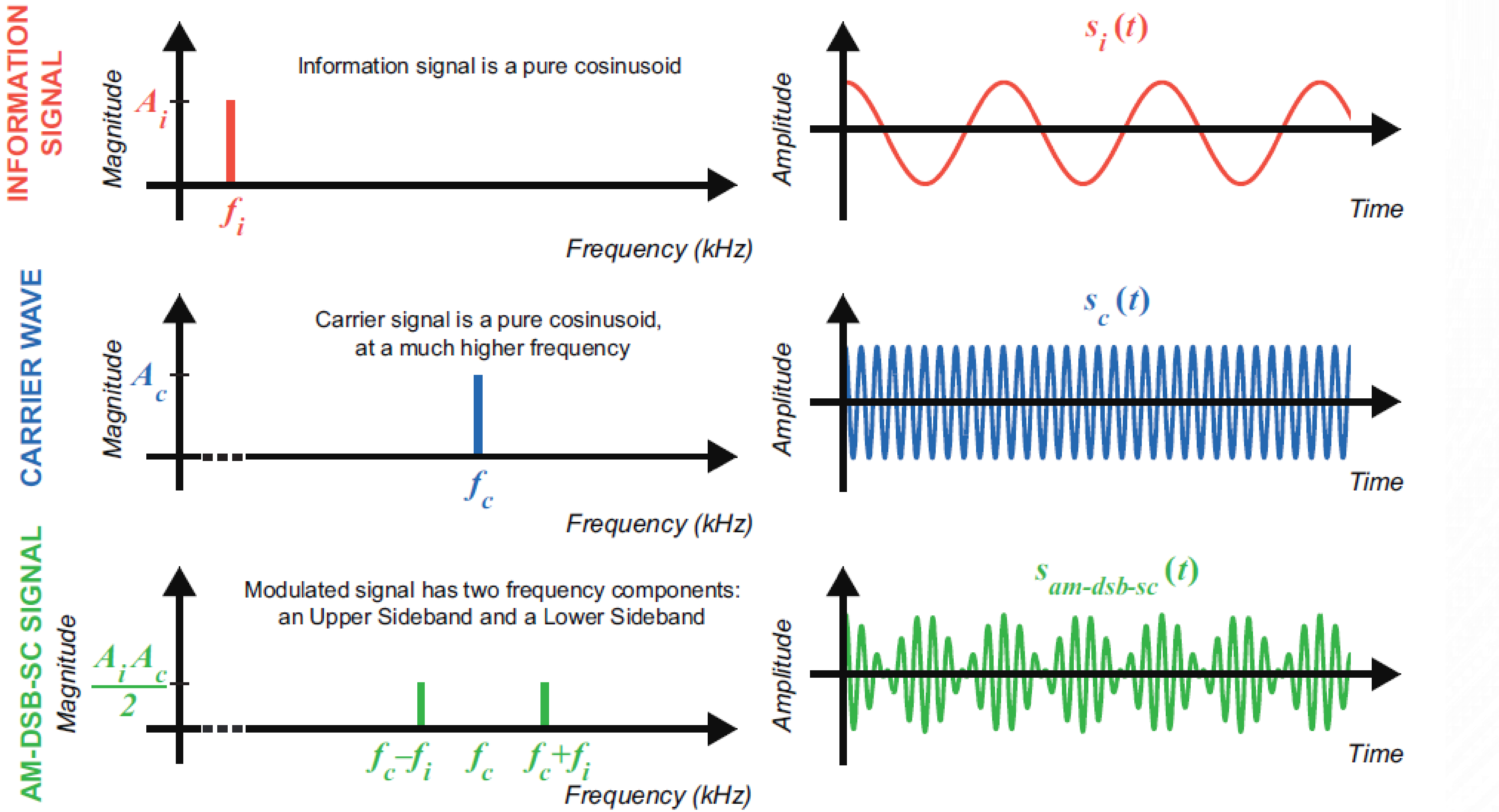
$$s_{am-dsb-sc}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t)$$

$$s_c(t) = A_c \cos(2\pi f_c t) = A_c \cos(\omega_c t)$$

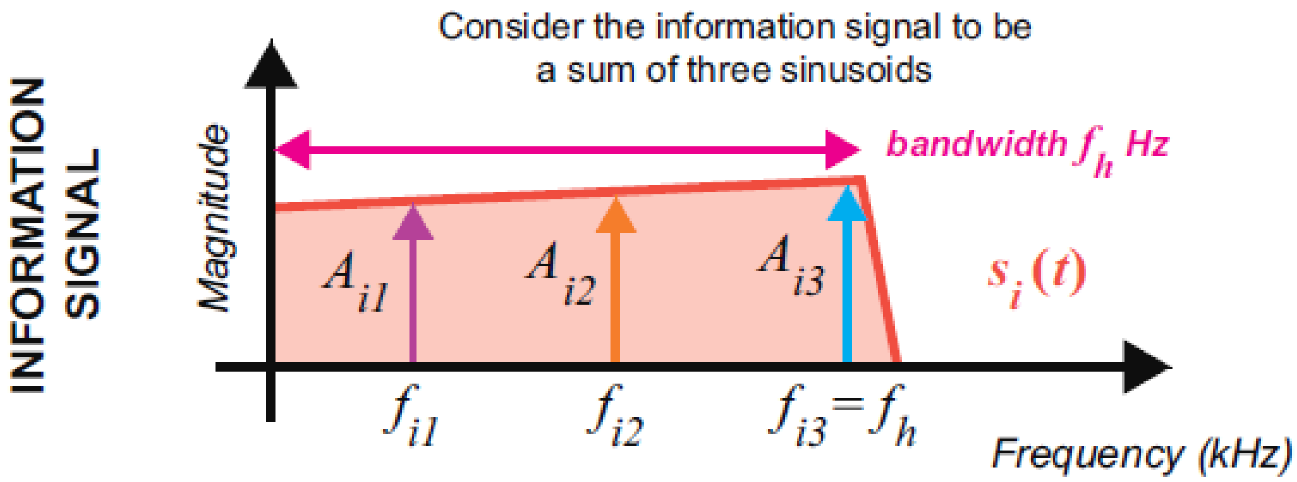
Pros: very simple implementation

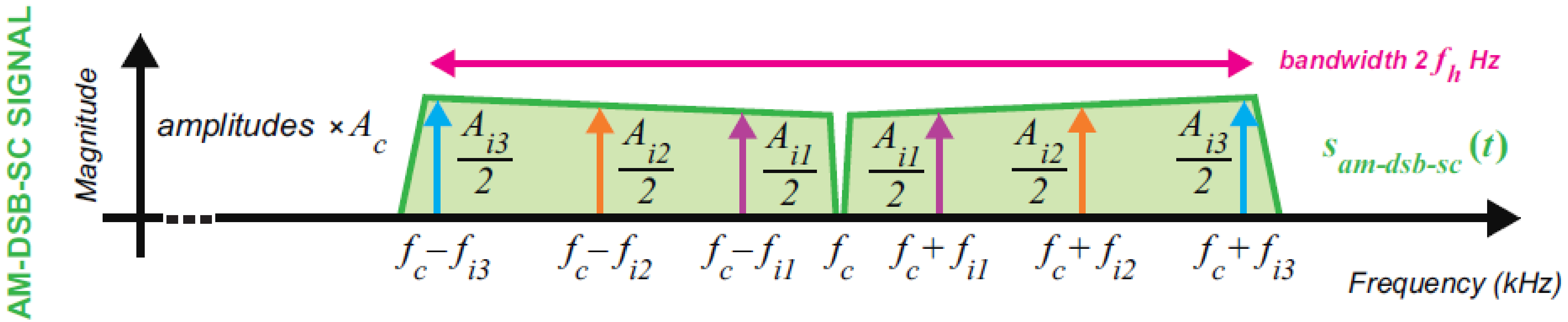
Cons: Require coherent demodulator, phase and frequency.

DSB-SC

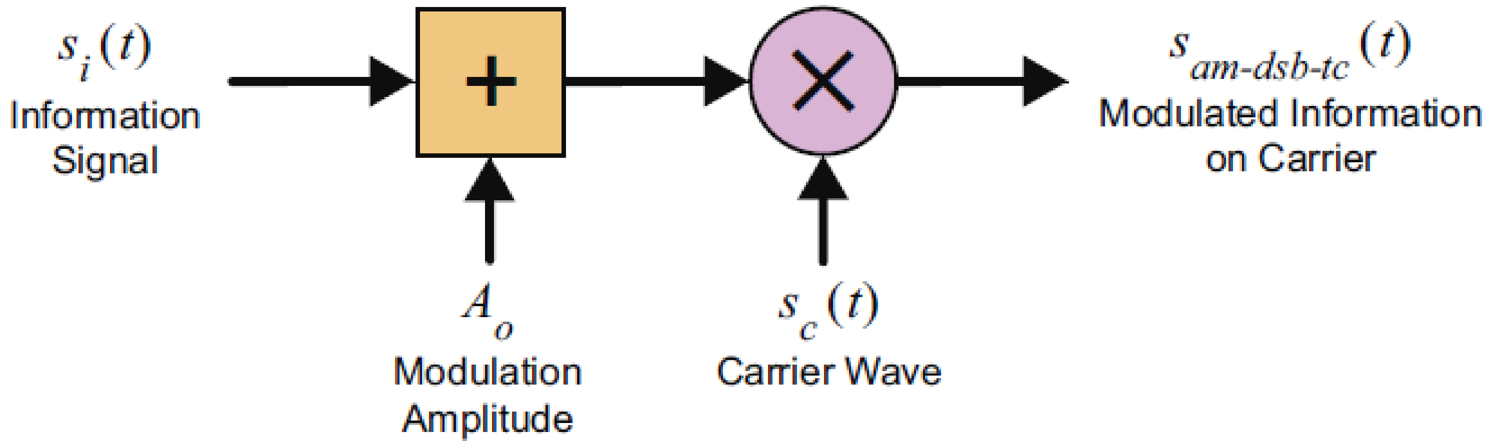

INFORMATION SIGNAL
Magnitude
A_i
f_i
Information signal is a pure cosinusoid
Frequency (kHz)
Amplitude
$s_i(t)$
Time
CARRIER WAVE
Magnitude
A_c
f_c
Carrier signal is a pure cosinusoid, at a much higher frequency
Frequency (kHz)
Amplitude
$s_c(t)$
Time
AM-DSB-SC SIGNAL
Magnitude
$\frac{A_i A_c}{2}$
Modulated signal has two frequency components: an Upper Sideband and a Lower Sideband
$f_c - f_i$
f_c
$f_c + f_i$
Frequency (kHz)
Amplitude
$s_{am-dsb-sc}(t)$
Time

DSB-SC BASE BAND





DSB-TC



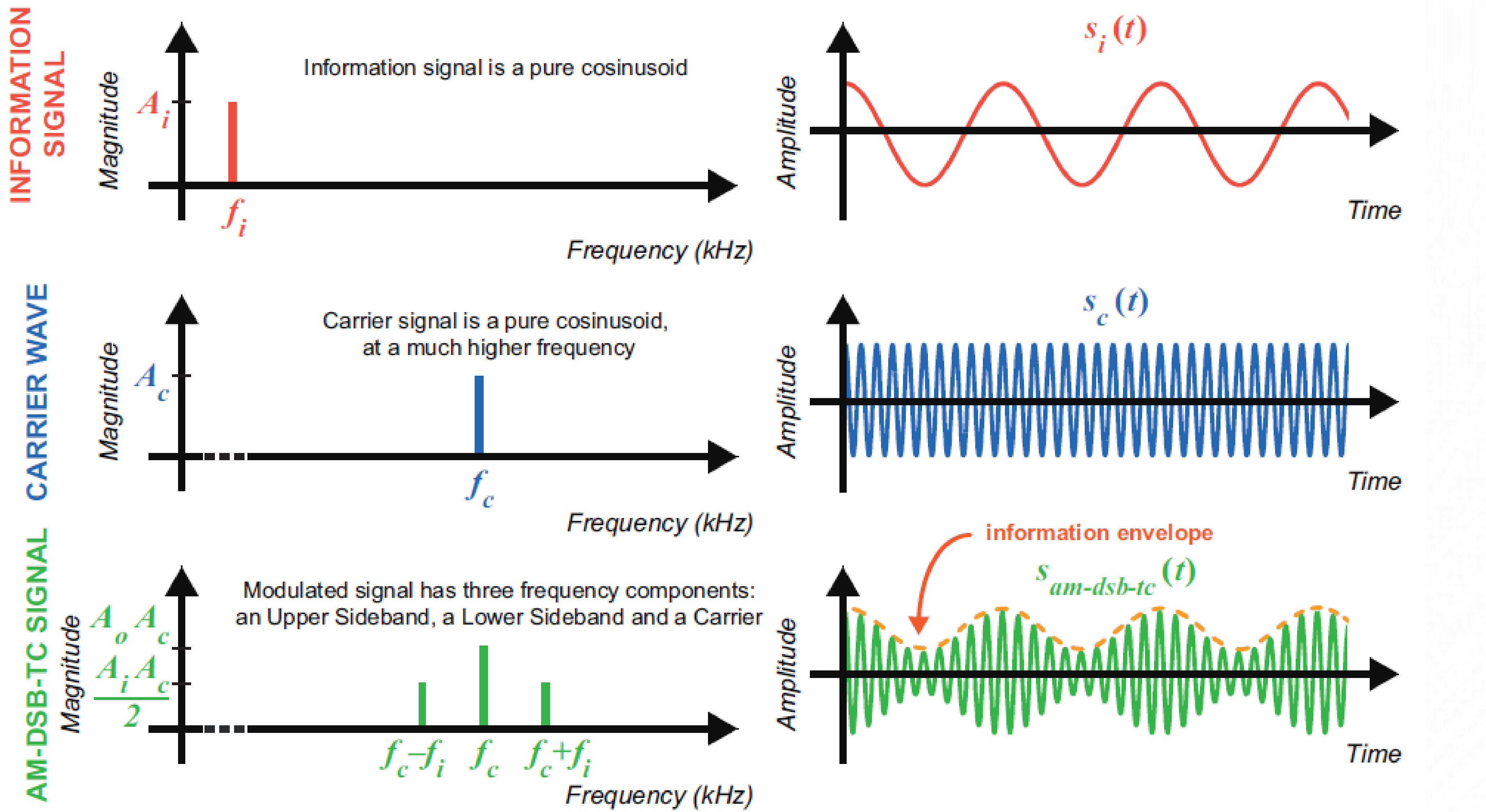
$$s_{am-dsb-tc}(t) = \left[A_o + s_i(t) \right] \times s_c(t)$$

$$s_{am-dsb-tc}(t) = \left[A_o + A_i \cos(\omega_i t) \right] A_c \cos(\omega_c t)$$

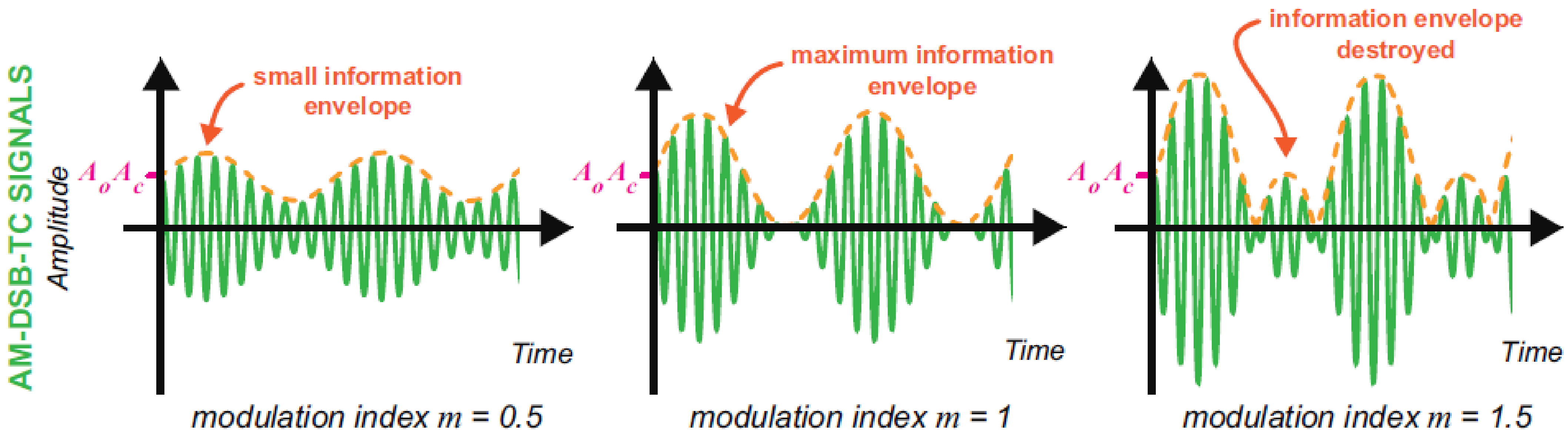
$$s_{am-dsb-tc}(t) = A_o \left[1 + m \cos(\omega_i t) \right] A_c \cos(\omega_c t)$$

m: modulation index

DSB-TC

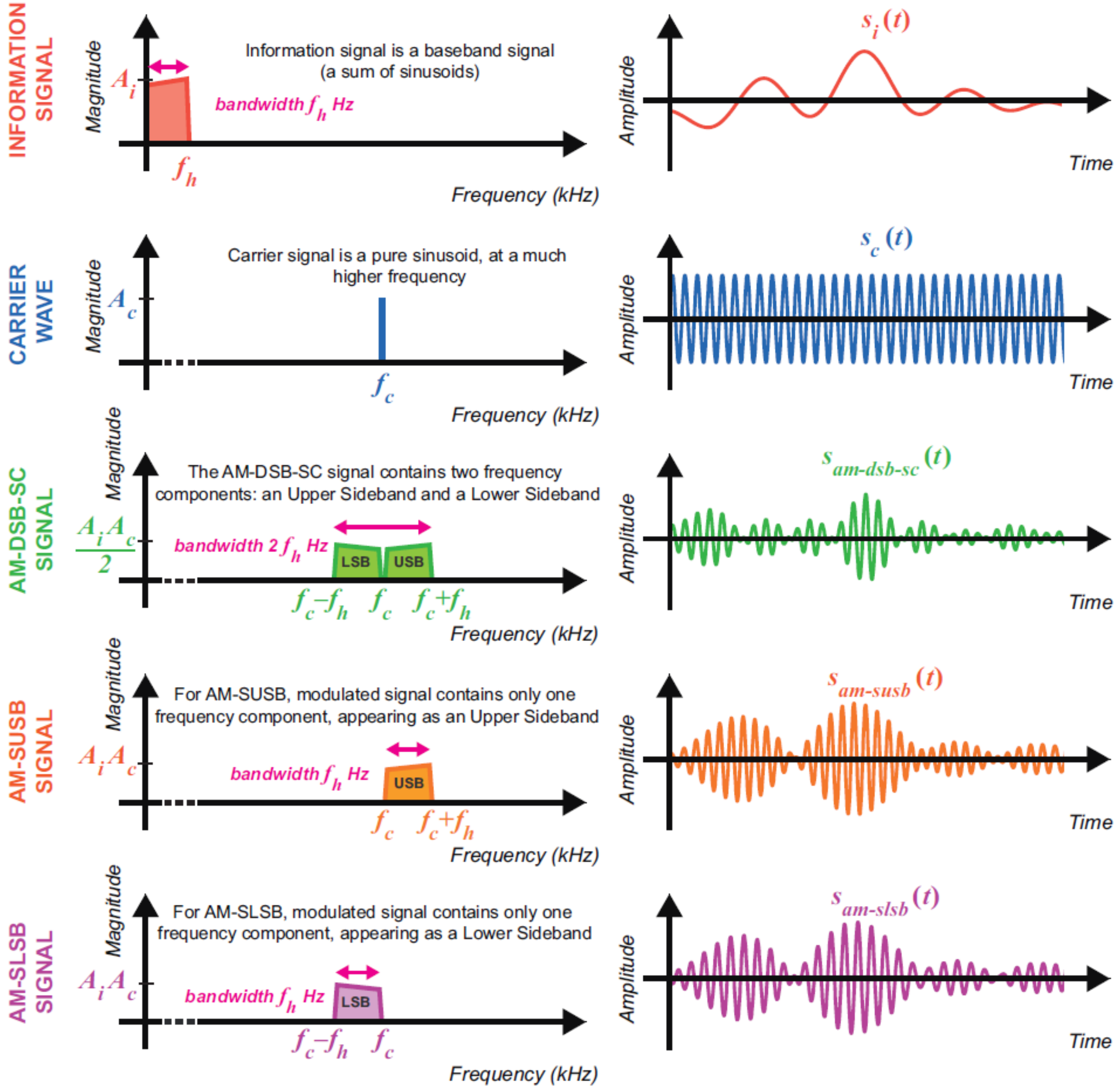

INFORMATION SIGNAL
Magnitude
A_i
f_i
Information signal is a pure cosinusoid
Frequency (kHz)
$s_i(t)$
Amplitude
Time
CARRIER WAVE
Magnitude
A_c
f_c
Carrier signal is a pure cosinusoid, at a much higher frequency
Frequency (kHz)
$s_c(t)$
Amplitude
Time
AM-DSB-TC SIGNAL
Magnitude
$A_o A_c$
$\frac{A_i A_c}{2}$
Modulated signal has three frequency components: an Upper Sideband, a Lower Sideband and a Carrier
$f_c - f_i$
f_c
$f_c + f_i$
Frequency (kHz)
information envelope
$s_{am\text{-}dsb\text{-}tc}(t)$
Amplitude
Time

MODULATION INDEX



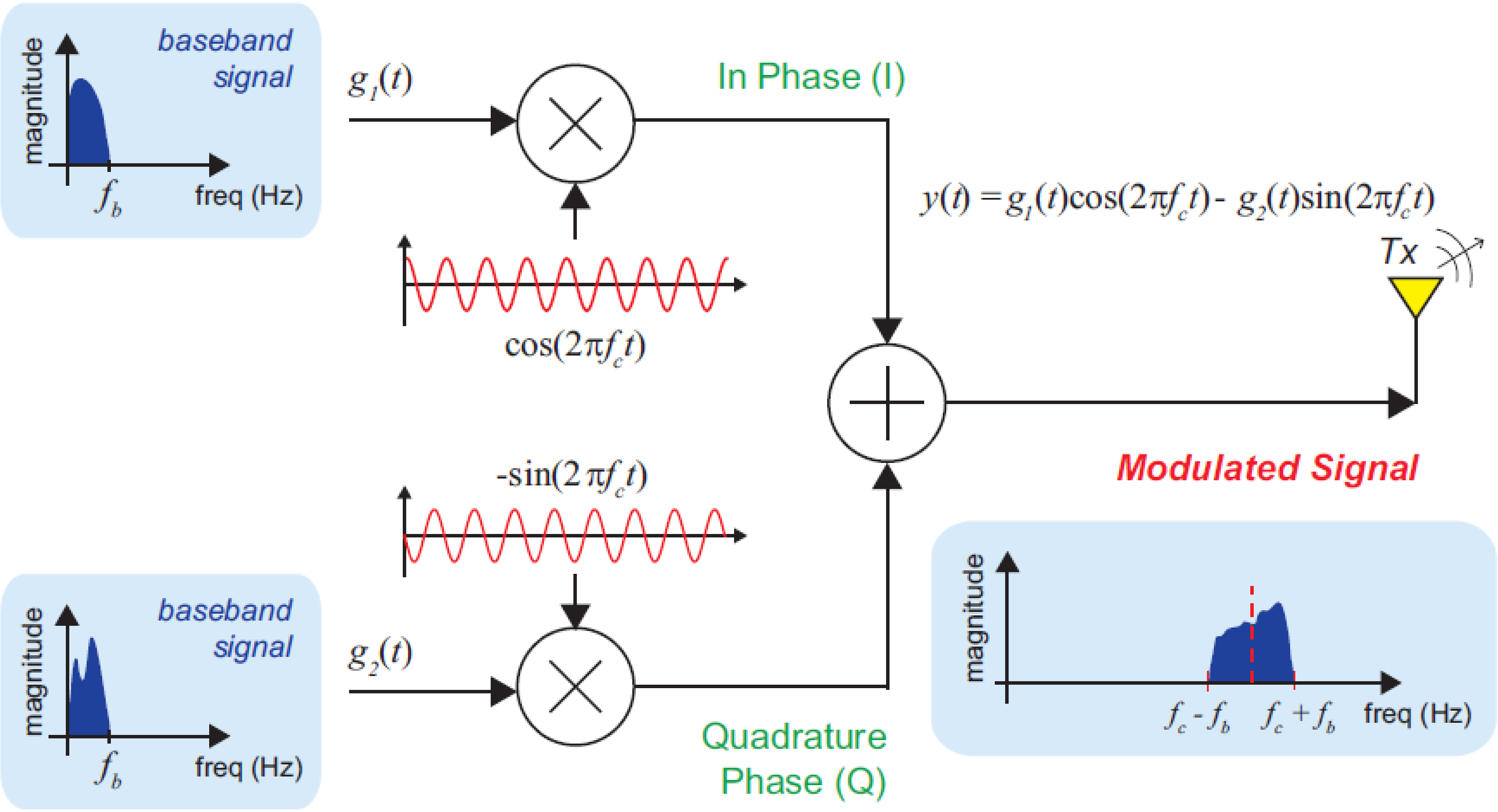
Pros: NOT require coherent demodulator
Cons: bad spectral efficiency

AM-SSB

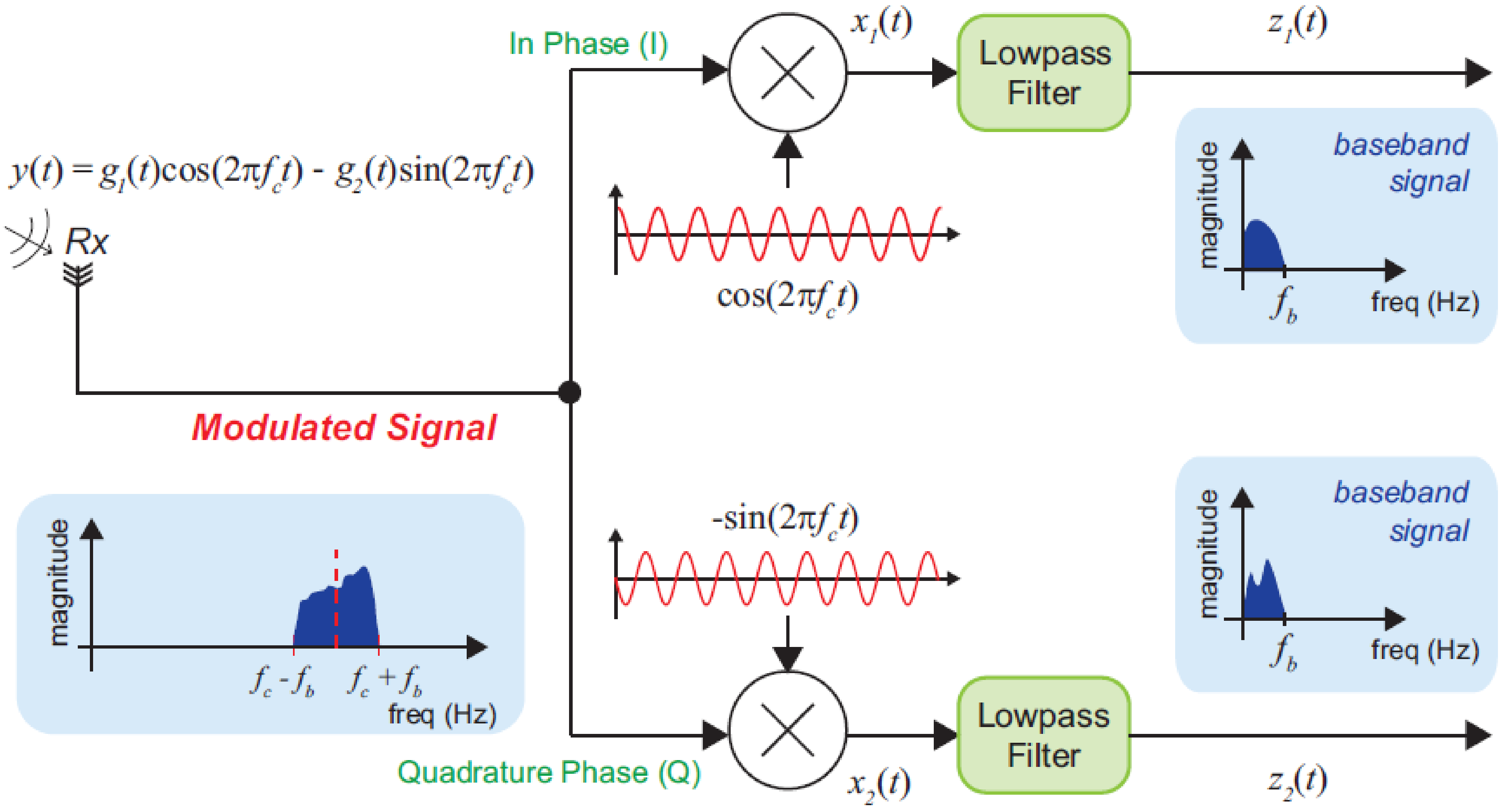

INFORMATION SIGNAL
Magnitude
Information signal is a baseband signal (a sum of sinusoids)
bandwidth f_h Hz
A_i
f_h
Frequency (kHz)
Amplitude
$s_i(t)$
Time
CARRIER WAVE
Carrier signal is a pure sinusoid, at a much higher frequency
A_c
f_c
$s_c(t)$
AM-DSB-SC SIGNAL
The AM-DSB-SC signal contains two frequency components: an Upper Sideband and a Lower Sideband
bandwidth 2 f_h Hz
$\frac{A_i A_c}{2}$
LSB
USB
$f_c - f_h$ f_c $f_c + f_h$
$s_{am\text{-}dsb\text{-}sc}(t)$
AM-SUSB SIGNAL
For AM-SUSB, modulated signal contains only one frequency component, appearing as an Upper Sideband
bandwidth f_h Hz
$A_i A_c$
f_c $f_c + f_h$
$s_{am\text{-}susb}(t)$
AM-SLSB SIGNAL
For AM-SLSB, modulated signal contains only one frequency component, appearing as a Lower Sideband
$f_c - f_h$ f_c
$s_{am\text{-}slsb}(t)$

QAM MODULATOR

- Advantages of QAM, we have better spectral efficiency.



QAM DEMODULATOR



I/Q PERFECT DEMO

$$x_1(t) = y(t)\cos(2\pi f_c t)$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$= g_1(t)\cos^2(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{1}{2} g_1(t)[1 + \cos(4\pi f_c t)] - \frac{1}{2} g_2(t)\sin(4\pi f_c t)$$

$$= \frac{1}{2} g_1(t) + \left[\frac{1}{2} g_1(t)\cos(4\pi f_c t) - \frac{1}{2} g_2(t)\sin(4\pi f_c t)\right]$$

lowpass filtered terms

I/Q PERFECT DEMO

$$x_2(t) = y(t)(-\sin(2\pi f_c t))$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right](-\sin(2\pi f_c t))$$

$$= [-g_1(t)\cos(2\pi f_c t)\sin(2\pi f_c t)] + g_2(t)\sin^2(2\pi f_c t)$$

$$= -\frac{1}{2}g_1(t)\sin(4\pi f_c t) + \frac{1}{2}g_2(t)[1 - \cos(4\pi f_c t)]$$

$$= \frac{1}{2}g_2(t) - \left[\frac{1}{2}g_1(t)\sin(4\pi f_c t) + \frac{1}{2}g_2(t)\cos(4\pi f_c t)\right]$$

lowpass filtered terms

PHASE SHIFT

$$x_1(t) = y(t)\cos(2\pi f_c t + \theta)$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t + \theta)$$

$$= g_1(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta) \quad - \quad g_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \theta)$$

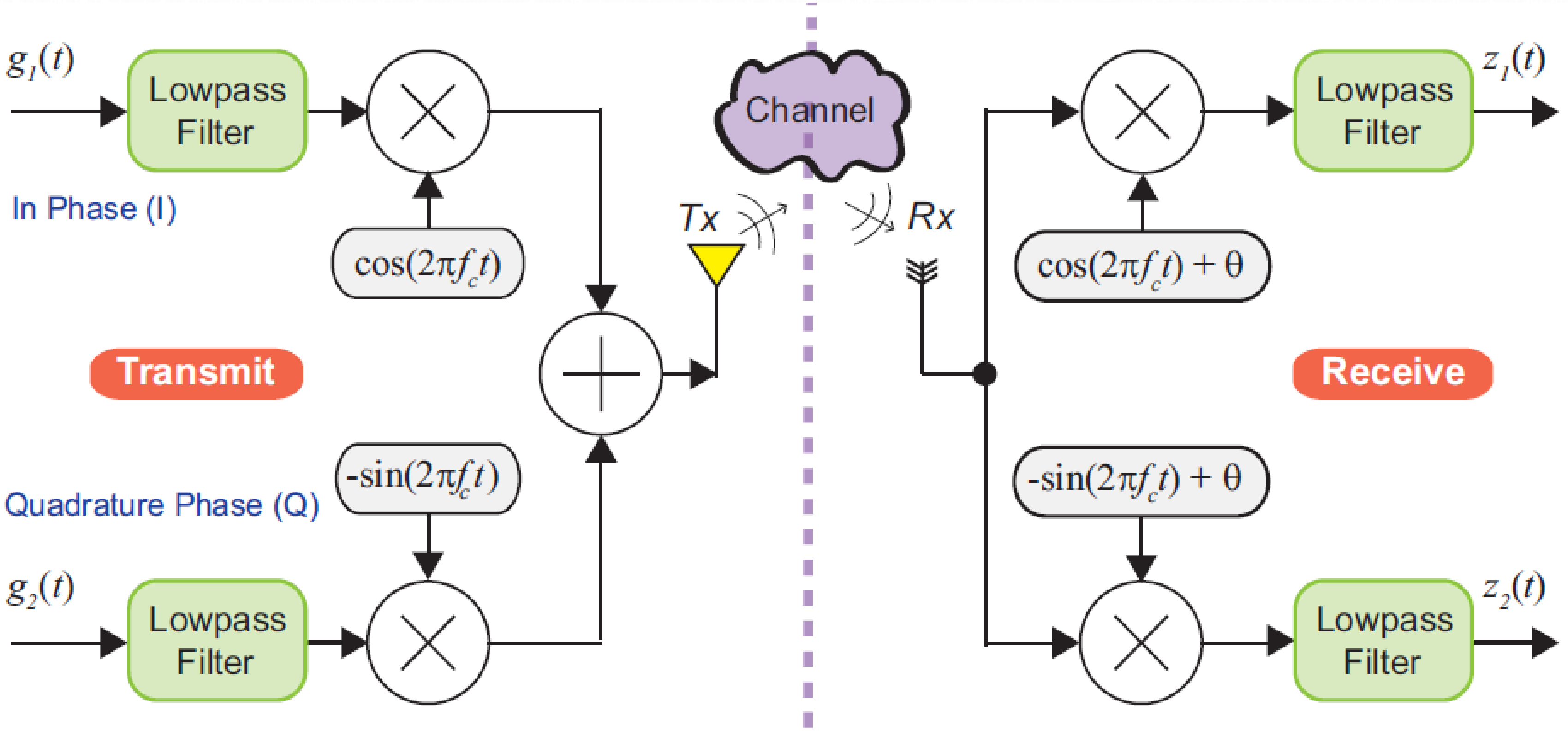
$$= \frac{1}{2}g_1(t)\left[\cos(-\theta) + \cos(4\pi f_c t + \theta)\right] - \frac{1}{2}g_2(t)\left[\sin(-\theta) + \sin(4\pi f_c t + \theta)\right]$$

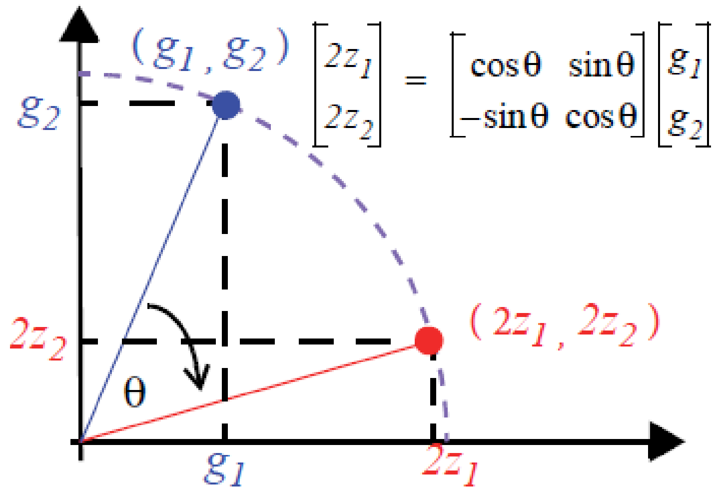
$$= \frac{1}{2}g_1(t)\left[\cos(\theta) + \cos(4\pi f_c t + \theta)\right] - \frac{1}{2}g_2(t)[-\sin(\theta) + \sin(4\pi f_c t + \theta)]$$

$$= \frac{1}{2}\left[g_1(t)\cos(\theta) + g_2(t)\sin(\theta)\right] + \left[\frac{1}{2}g_1(t)\cos(4\pi f_c t + \theta) - \frac{1}{2}g_2(t)\sin(4\pi f_c t + \theta)\right]$$

lowpass filtered terms

PHASE OFFSET

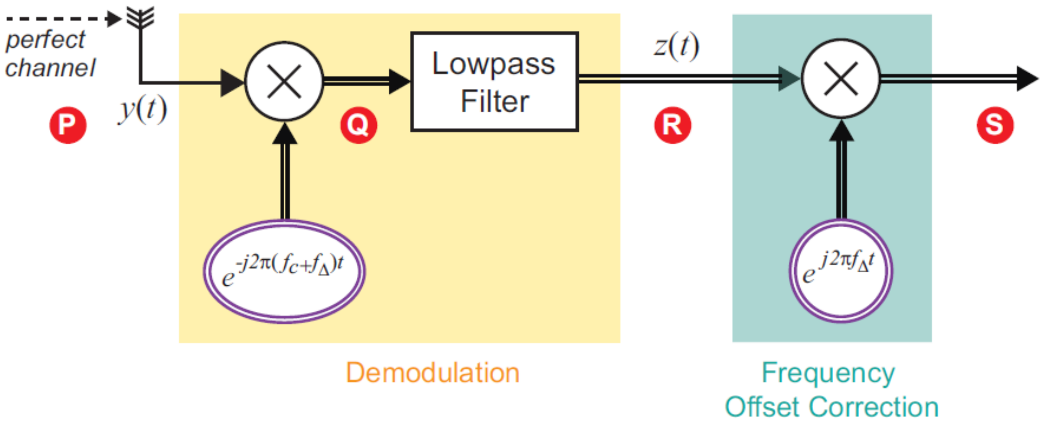


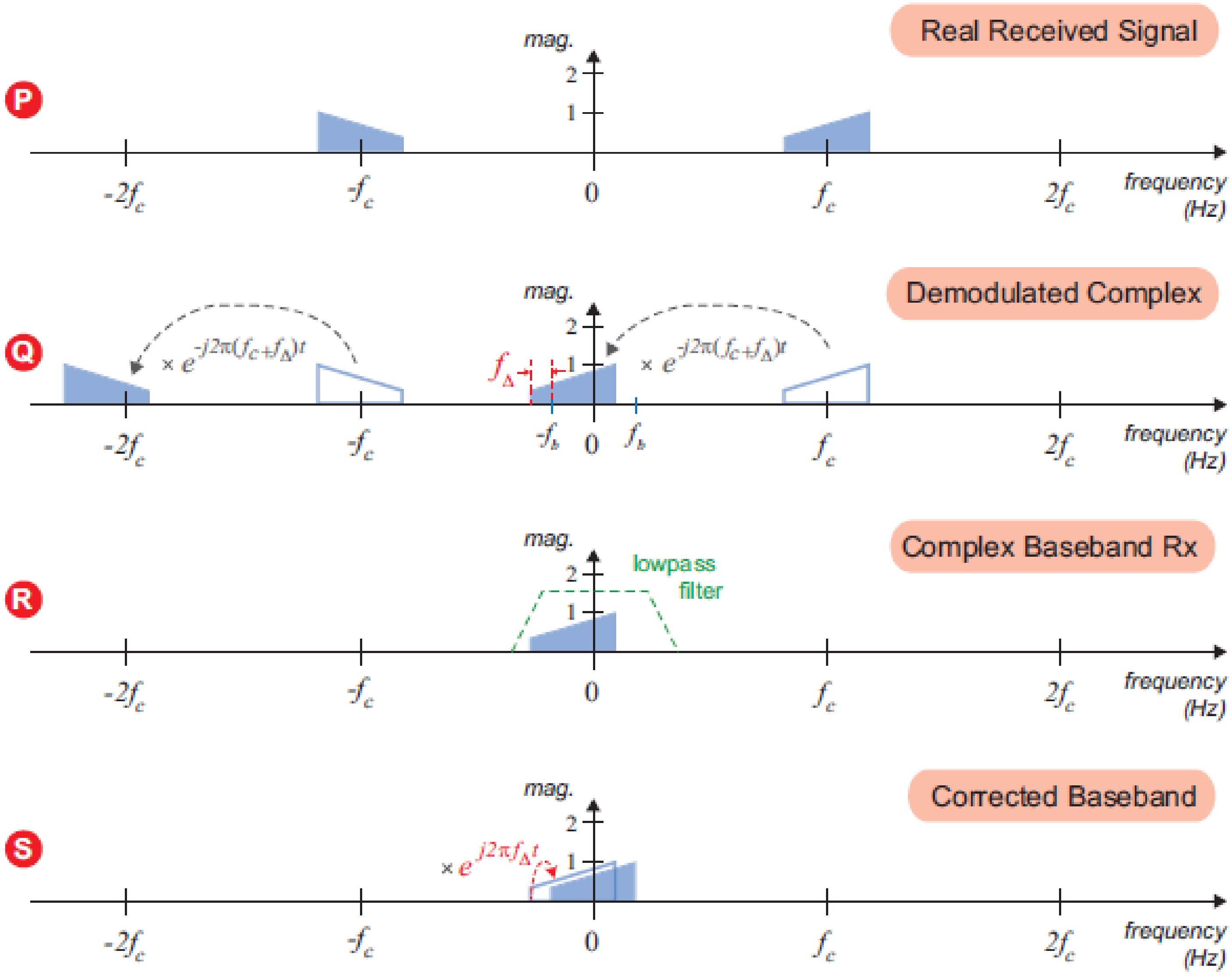


$$z_1(t) = 0.5[g_1(t)\cos(\theta) + g_2(t)\sin(\theta)]$$

$$z_2(t) = 0.5[-g_1(t)\sin(\theta) + g_2(t)\cos(\theta)]$$

FREQUENCY OFFSET

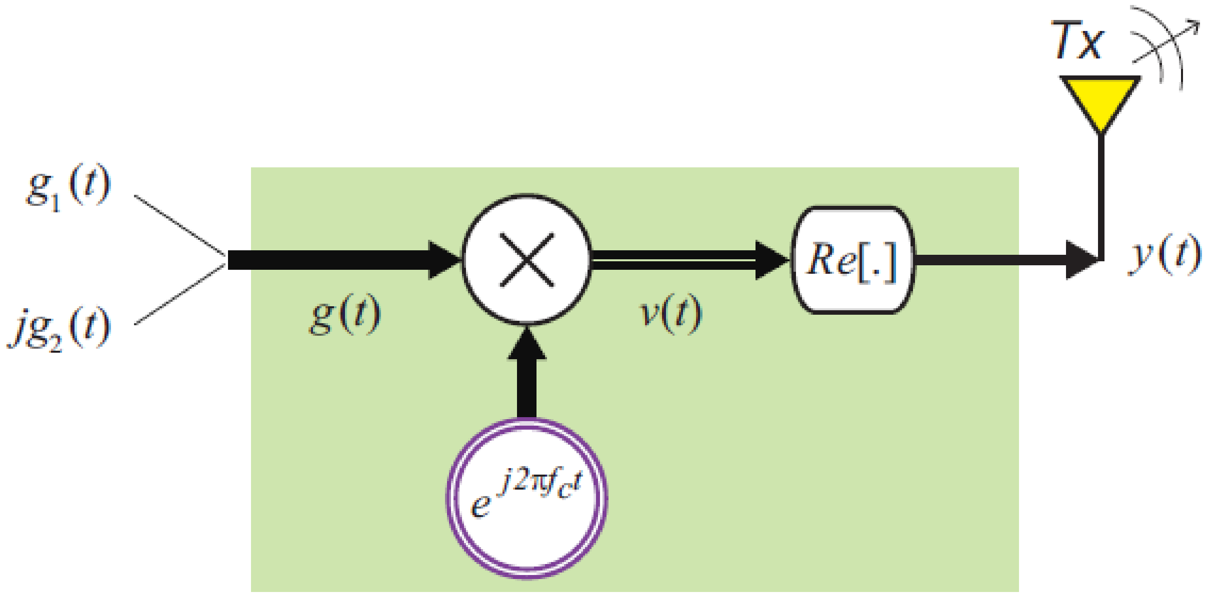




WE HAVE TO CORRECT OFFSET BEFORE START WORKING WITH SDR

COMPLEX EXPONENTIAL

$$g(t) = g_1(t) + jg_2(t) \qquad e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$



COMPLEX EXPONENTIAL

$$v(t) = g(t)e^{j2\pi f_c t} = \left[g_1(t) + jg_2(t)\right]e^{j2\pi f_c t}$$

$$= \left[g_1(t) + jg_2(t)\right]\left[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right]$$

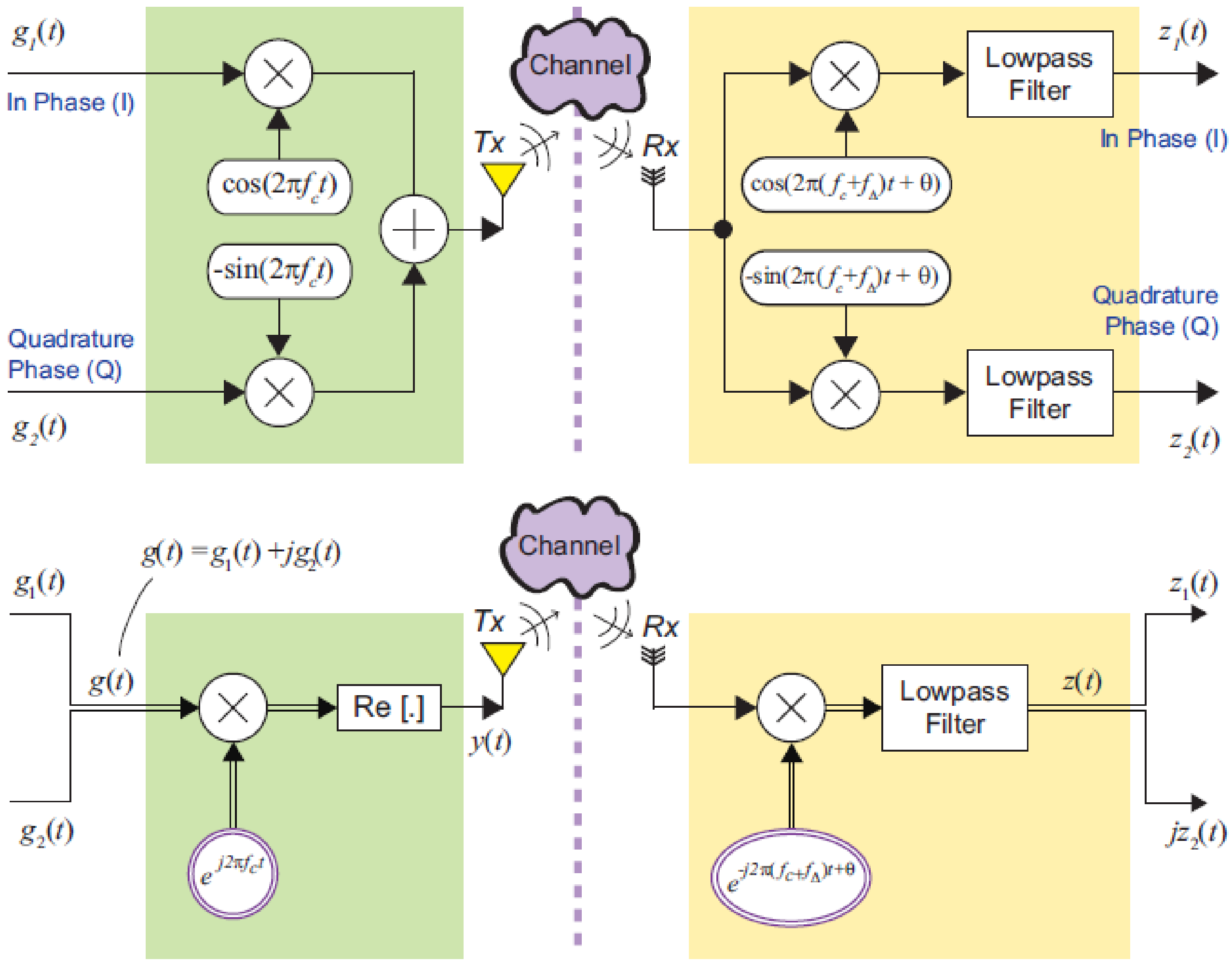
$$= g_1(t)\cos(2\pi f_c t) + jg_2(t)\cos(2\pi f_c t) + jg_1(t)\sin(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)$$

$$= \underbrace{\left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right]}_{\textbf{Real}} + \underbrace{j\left[g_1(t)\sin(2\pi f_c t) + g_2(t)\cos(2\pi f_c t)\right]}_{\textbf{Imaginary}}$$

$$\Re e\{v(t)\} = \Re e\left\{\left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right] + j\left[g_1(t)\sin(2\pi f_c t) + g_2(t)\cos(2\pi f_c t)\right]\right\}$$

$$= g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)$$

REAL HARDWARE VS COMPLEX NOTATION



THEORETICAL DEMODULATION ON DSB-SC

- The simplest receiver will be a DSB-SC, because we just need to multiply by the carrier.
- This will works ONLY if no phase or frequency offset.

$$s_{am-dsb-sc}(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t \right)$$

$$s_d(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t \right)\cos(\omega_c t)$$

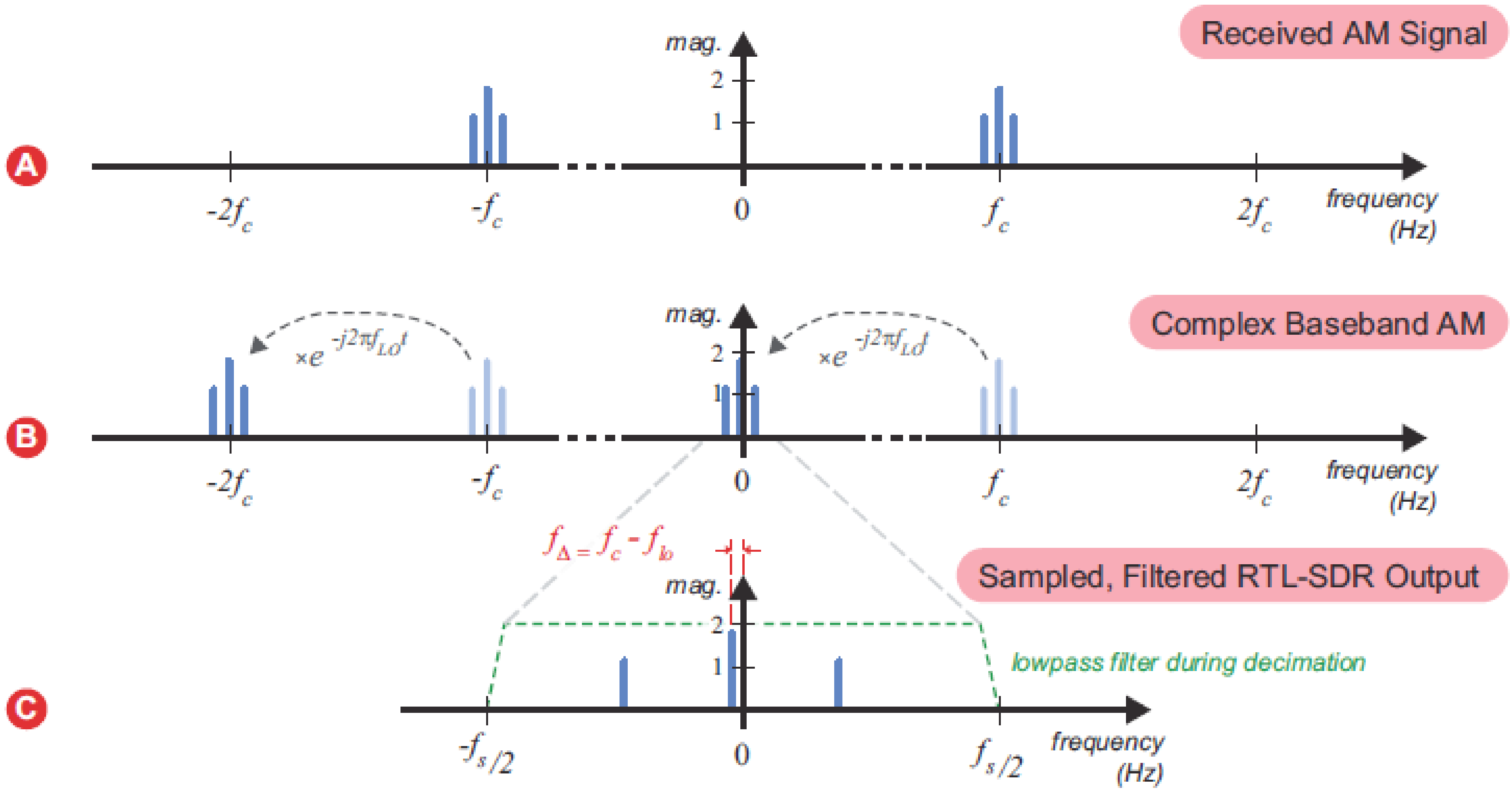
$$= \frac{A_i A_c}{2}\left[\cos\big((\omega_c - \omega_i)t\big)\cos(\omega_c t) + \cos\big((\omega_c + \omega_i)t\big)\cos(\omega_c t) \right]$$

$$= \frac{A_i A_c}{2}\left[\begin{array}{l} \frac{1}{2}\cos(-\omega_i t) + \frac{1}{2}\cos\big((2\omega_c - \omega_i)t\big) \\ + \frac{1}{2}\cos(\omega_i t) + \frac{1}{2}\cos\big((2\omega_c + \omega_i)t\big) \end{array} \right].$$

(lowpass filtered)

$$s_d(t) = \frac{A_i A_c}{4}\cos(-\omega_i t) + \frac{A_i A_c}{4}\cos(\omega_i t) = \frac{A_i A_c}{2}\cos(\omega_i t)$$

REAL COHERENT DEMODULATOR DSB-TC



$$s_{am\mathrm{RF}}(t) = s_{am-dsb-tc}(t) = A_o A_c \cos(\omega_c)t + \frac{A_i A_c}{2}\cos(\omega_c - \omega_i)t + \frac{A_i A_c}{2}\cos(\omega_c + \omega_i)t$$

REAL COHERENT DEMODULATOR

$$s_{bband}(t) = s_{am\mathrm{RF}}(t)e^{-j\omega_{lo}t}$$

$$= s_{am\mathrm{RF}}(t)\times\left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

$$= A_o A_c \cos(\omega_c t) \times\left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

$$+ \frac{A_i A_c}{2}\left(\cos(\omega_c t - \omega_i t) + \cos(\omega_c t + \omega_i t)\right)\times\left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

$$s_{bband}(t) = \frac{A_o A_c}{2}\Big[\cos(\omega_c t - \omega_{lo}t) + \cos(\omega_c t + \omega_{lo}t)$$

$$- j\sin(\omega_c t - \omega_{lo}t) - j\sin(\omega_c t + \omega_{lo}t)\Big]$$

$$+ \frac{A_i A_c}{4}\Big[\cos(\omega_c t - \omega_i t - \omega_{lo}t) + \cos(\omega_c t - \omega_i t + \omega_{lo}t)$$

$$+ \cos(\omega_c t + \omega_i t - \omega_{lo}t) + \cos(\omega_c t + \omega_i t + \omega_{lo}t)$$

$$- j\sin(\omega_c t - \omega_i t - \omega_{lo}t) - j\sin(\omega_c t - \omega_i t + \omega_{lo}t)$$

$$- j\sin(\omega_c t + \omega_i t - \omega_{lo}t) - j\sin(\omega_c t + \omega_i t + \omega_{lo}t)\Big].$$

PERFECT DEMODULATOR

- Real demodulator, w/offset

$$s_{\text{RTL-SDR}}(t) = \frac{A_o A_c}{2}\left[\cos(\omega_c t - \omega_{lo} t) - j\sin(\omega_c t - \omega_{lo} t)\right]$$
$$+ \frac{A_i A_c}{4}\left[\cos(\omega_c t - \omega_{lo} t - \omega_i t) + \cos(\omega_c t - \omega_{lo} t + \omega_i t)\right.$$
$$\left. - j\sin(\omega_c t - \omega_{lo} t - \omega_i t) - j\sin(\omega_c t - \omega_{lo} t + \omega_i t)\right].$$

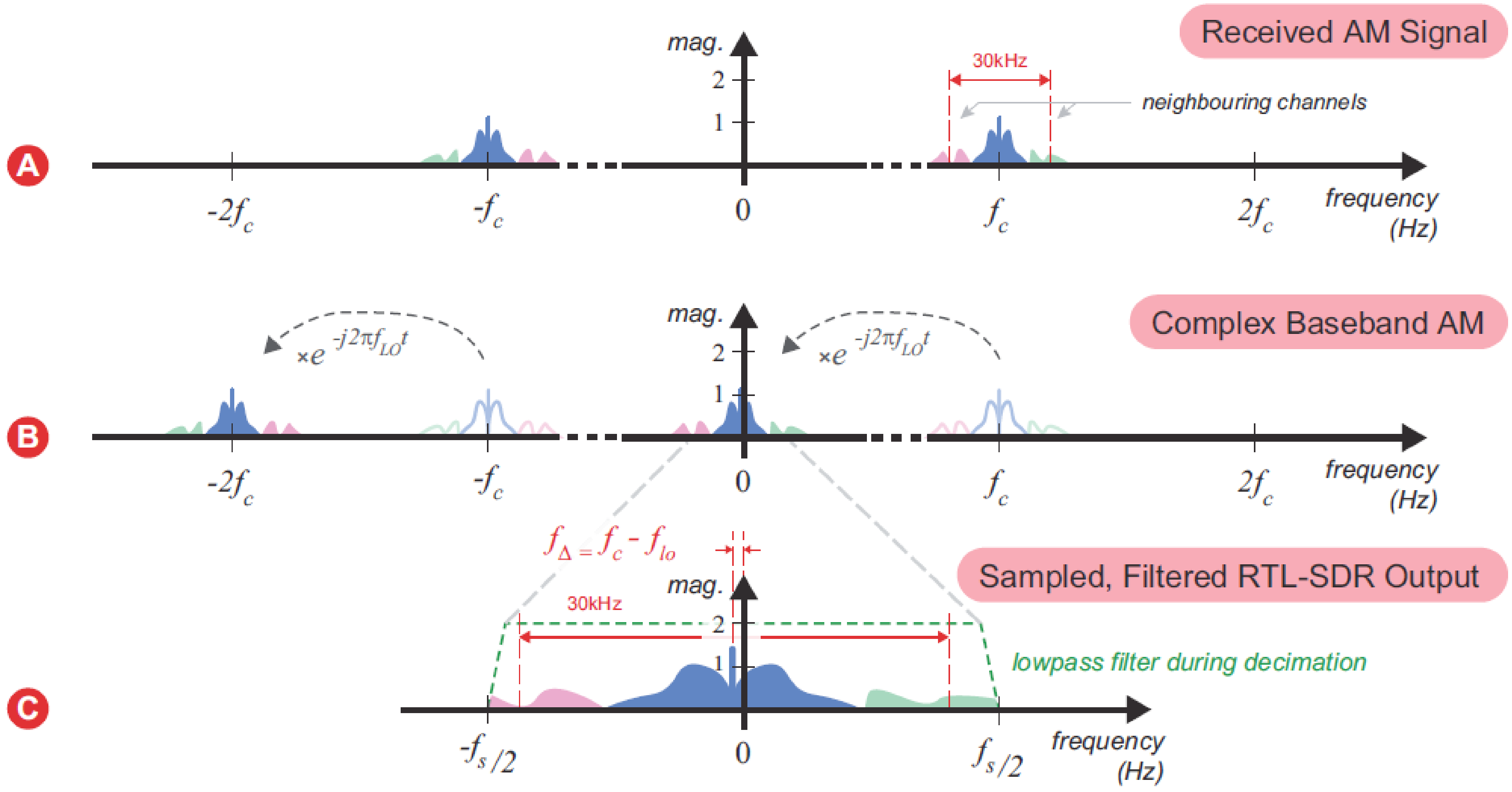
- Perfect demo

$$s_{\text{RTL-SDR}}(t) = \frac{A_o A_c}{2}\left[\cos(0) - j\sin(0)\right]$$
$$+ \frac{A_i A_c}{4}\left[\cos(-\omega_i t) + \cos(\omega_i t) - j\sin(-\omega_i t) - j\sin(\omega_i t)\right]$$

$$s_{ip}(t) = \Re e\left[s_{\text{RTL-SDR}}(t)\right] = \frac{A_o A_c}{2} + \frac{A_i A_c}{4}\left[\cos(-\omega_i t) + \cos(\omega_i t)\right] \quad s_{qp}(t) = \Im m\left[s_{\text{RTL-SDR}}(t)\right] = -j\frac{A_i A_c}{4}\left[\sin(-\omega_i t) + \sin(\omega_i t)\right]$$

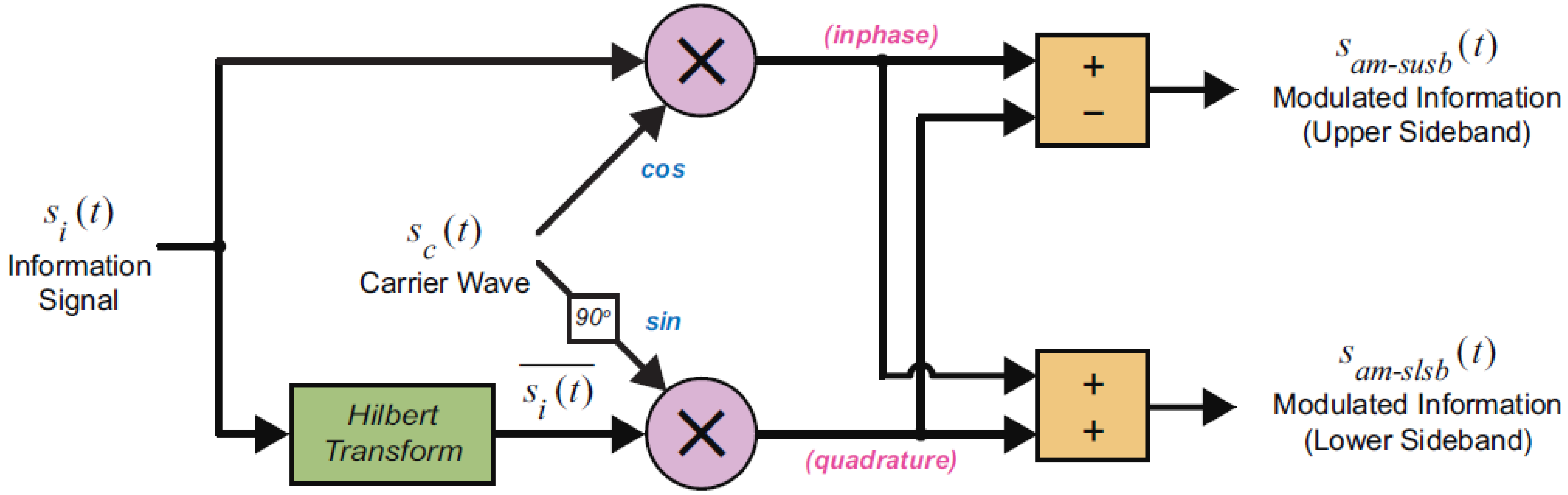
- Because is hard to get "**perfect tuning**" on the carrier, we need a coherent receiver with **frequency and phase recovery** to avoid this problems.

DEMODULATED RECEIVED SPECTRUM



GENERATION AM-SSB

EASY way is to filter AM-DSB-SC, BUT filter needs very large attenuation and rolloff factor



$$s_{am-ssb}(t) = s_i(t)\,\Re e\left[s_c(t)\right] \mp \overline{s_i(t)}\,\Im m\left[s_c(t)\right]$$

SSB TRANSMITTER

$$s_i(t) \rightarrow \overline{s_i(t)}$$

$$A_i \cos(\omega_i t) \rightarrow A_i \sin(\omega_i t)$$

$$s_c(t) = A_c \cos(2\pi f_c t) + A_c \sin(2\pi f_c t)$$

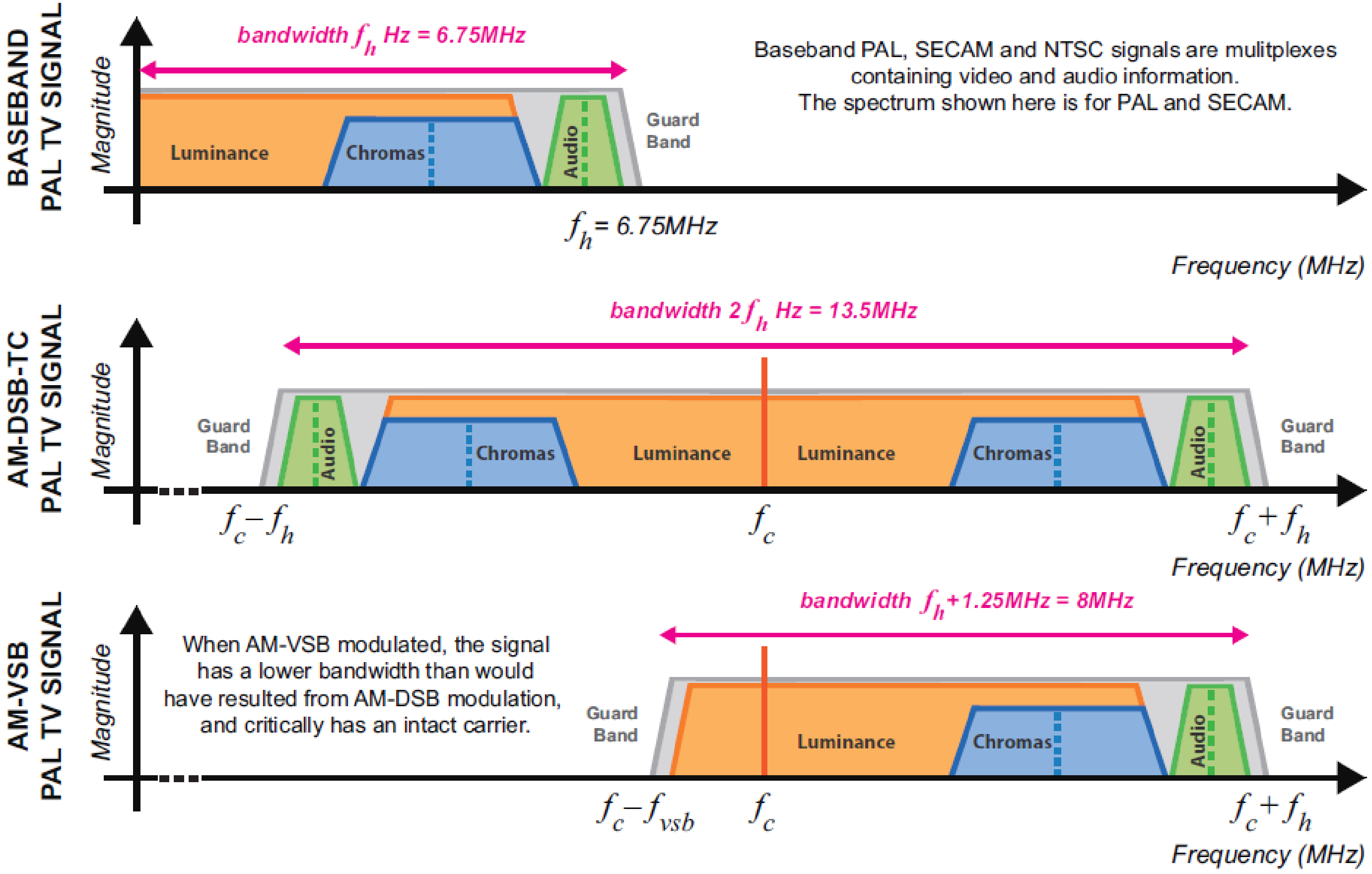
$$s_{am-ssb}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t) \mp A_i \sin(\omega_i t) A_c \sin(\omega_c t)$$

$$s_{am-susb}(t) = \frac{A_i A_c}{2}\Big(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\Big) - \frac{A_i A_c}{2}\Big(\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t\Big) = A_i A_c \cos(\omega_c + \omega_i)t$$

$$s_{am-slsb}(t) = \frac{A_i A_c}{2}\Big(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\Big) + \frac{A_i A_c}{2}\Big(\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t\Big) = A_i A_c \cos(\omega_c - \omega_i)t \ .$$

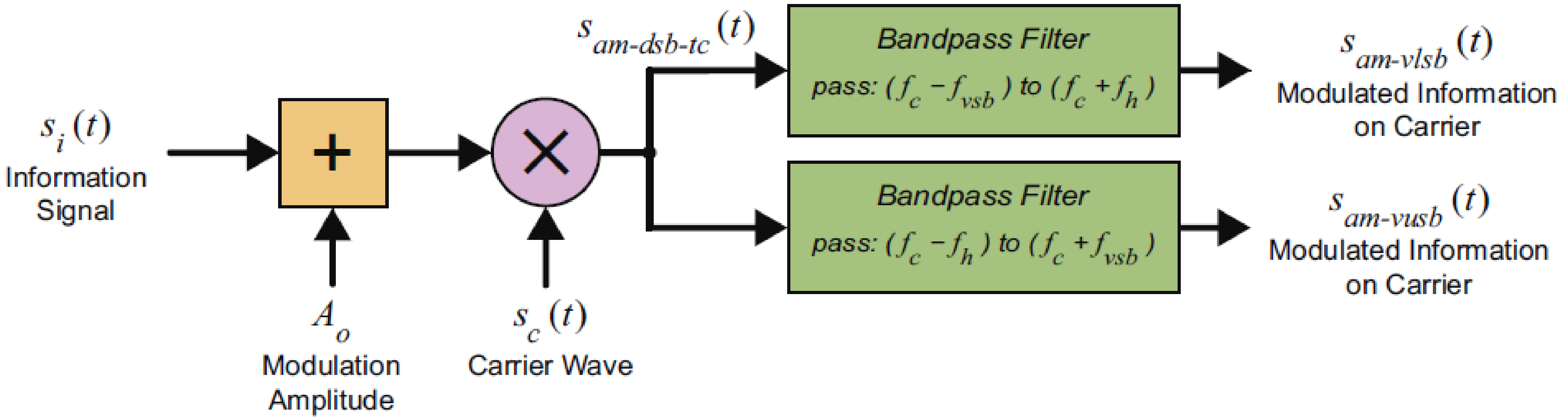
VESTIGIAL SIDE BAND (VSB)

- It was created to reduce the bandwidth on analog TV channels that use DSB-TC.
- The modulator is DSB-TC, but with a pass-band filter at the output. This filter allows to pass the carrier plus a portion of the lower band, the vestigial band.



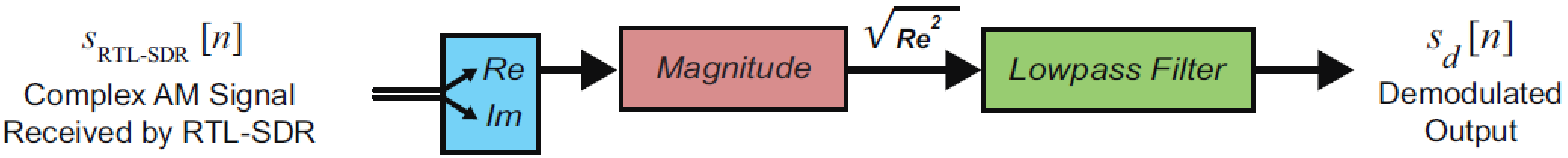
VSB

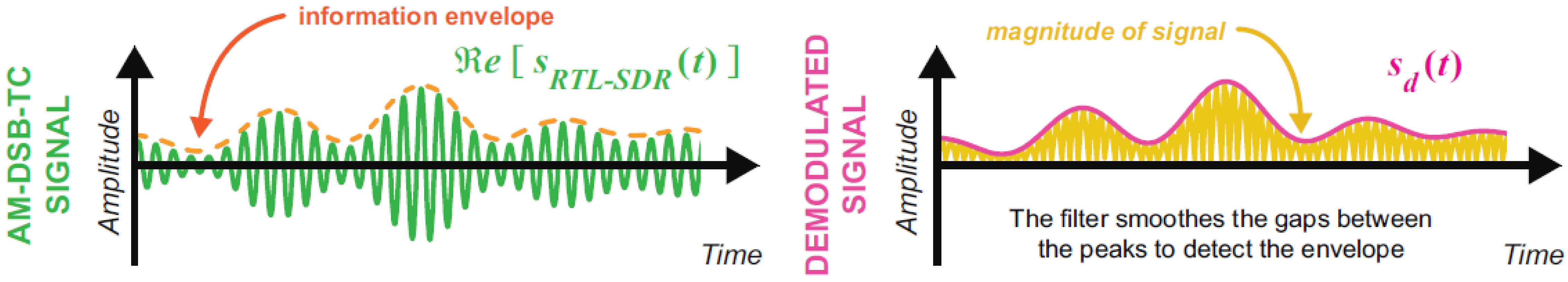
- Transmitter



NON-COHERENT DEMO: ENVELOPE DETECTOR

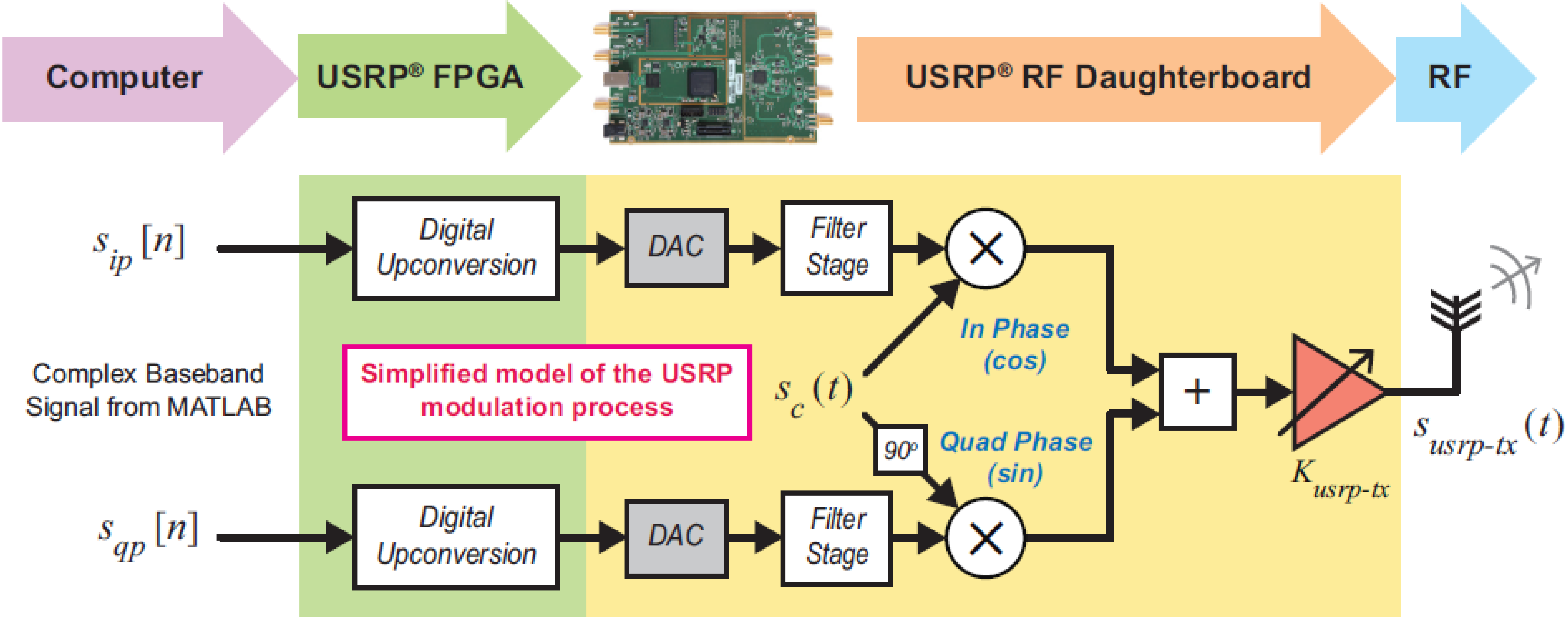
- Classical detector, only works for DSB-TC



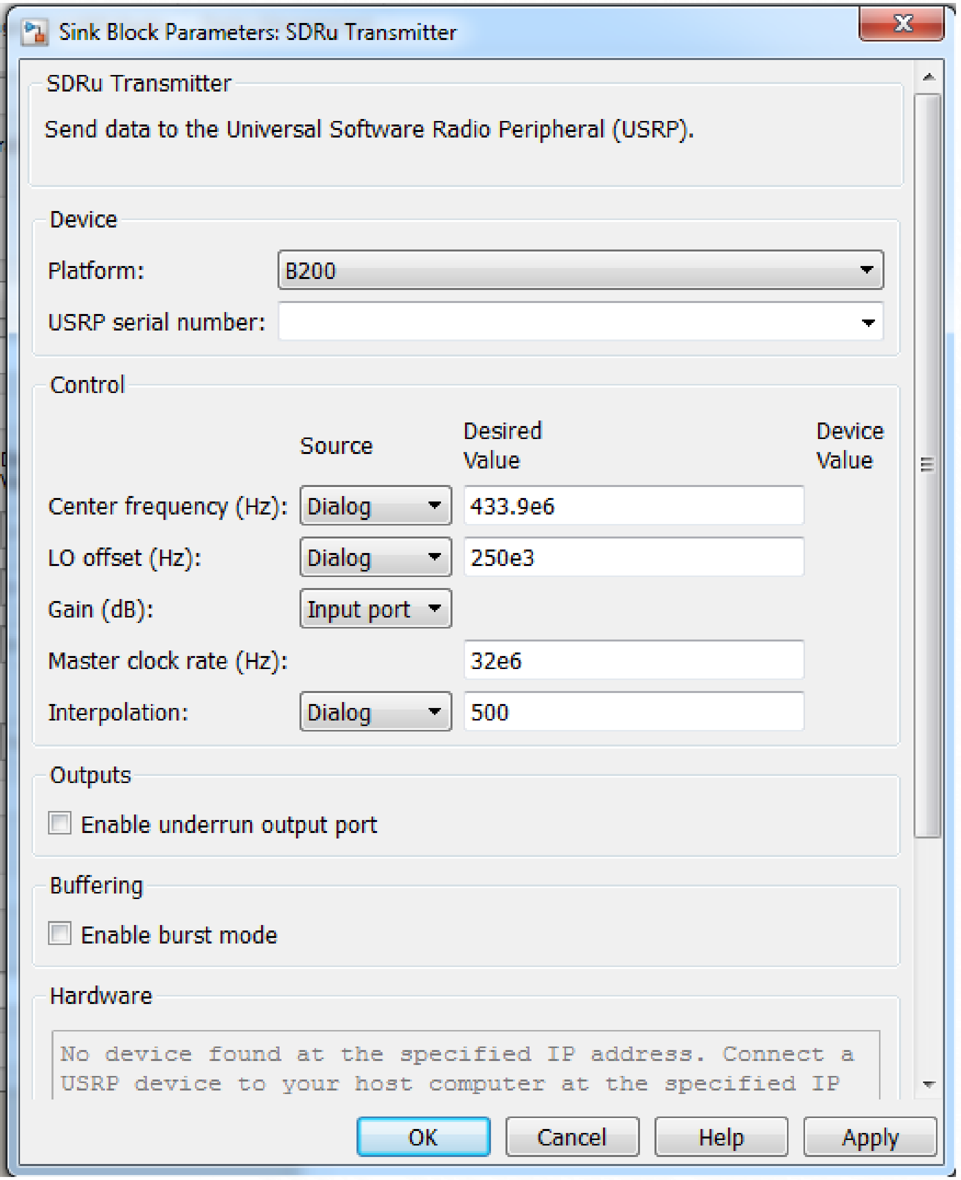


USRP TX

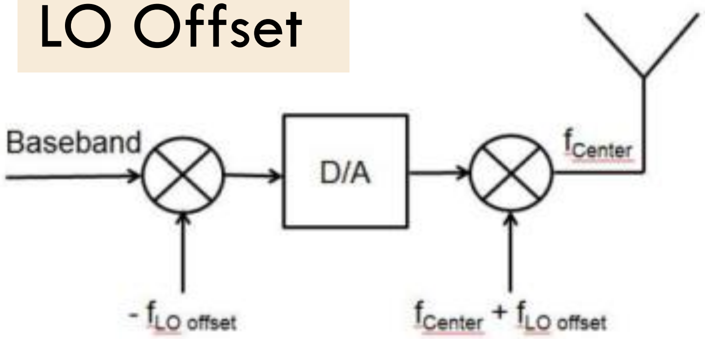
- USRP hardware support package is required.
- By default only digital up-conversion is done on the FGPA



USRP MATLAB PARAMETERS

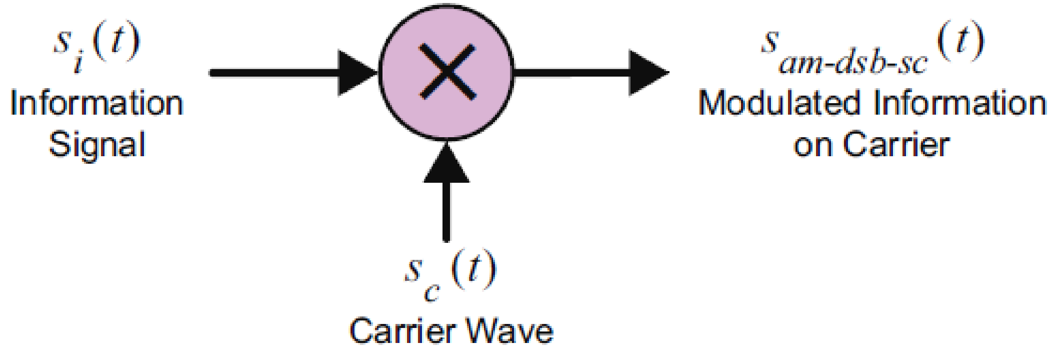


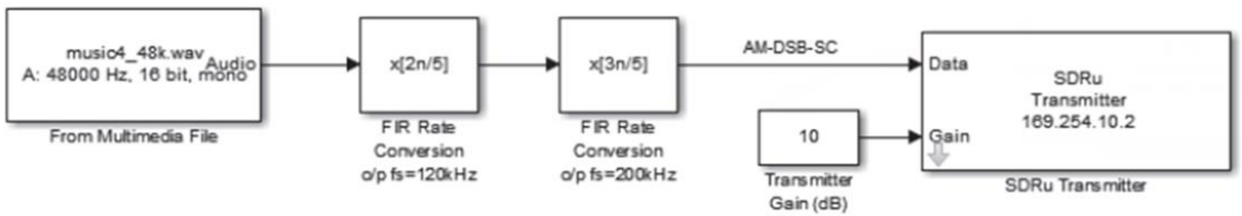
LO Offset

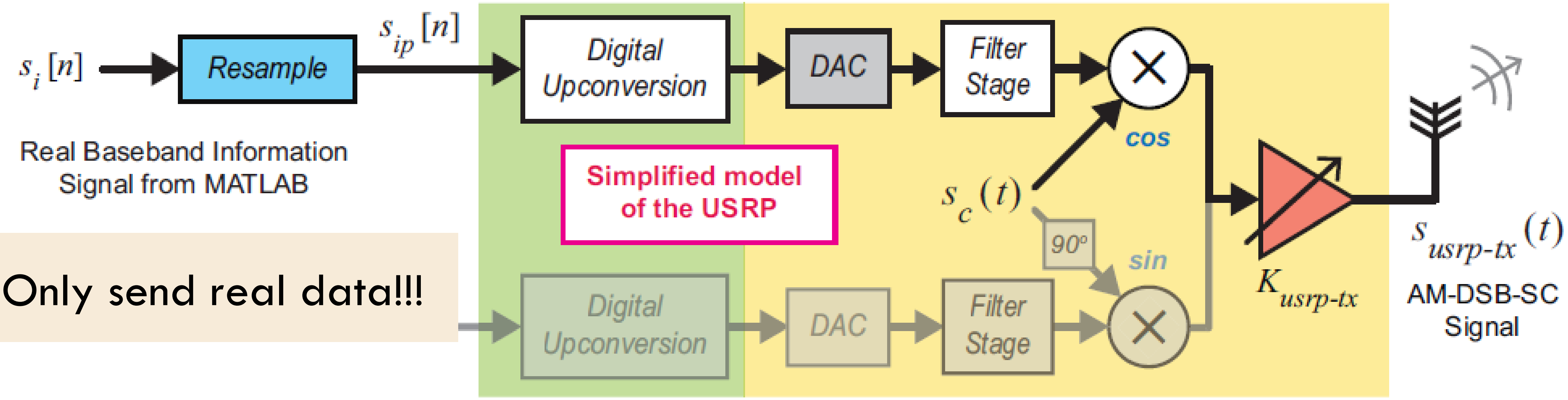


- TX Gain: expressed as scalar. Max 90dB
- RX Gain: 73dB
- Clock Freq: 5Mhz a 56Mhz
- DUC: default 512, correspond to simple rate of 1e8/512=195Khz
- Underrun: “+” not enough samples, “0” no data loss
- Enable burst: no underrun /overrun, fixed number of frames

USRP TX AM DSB-SC

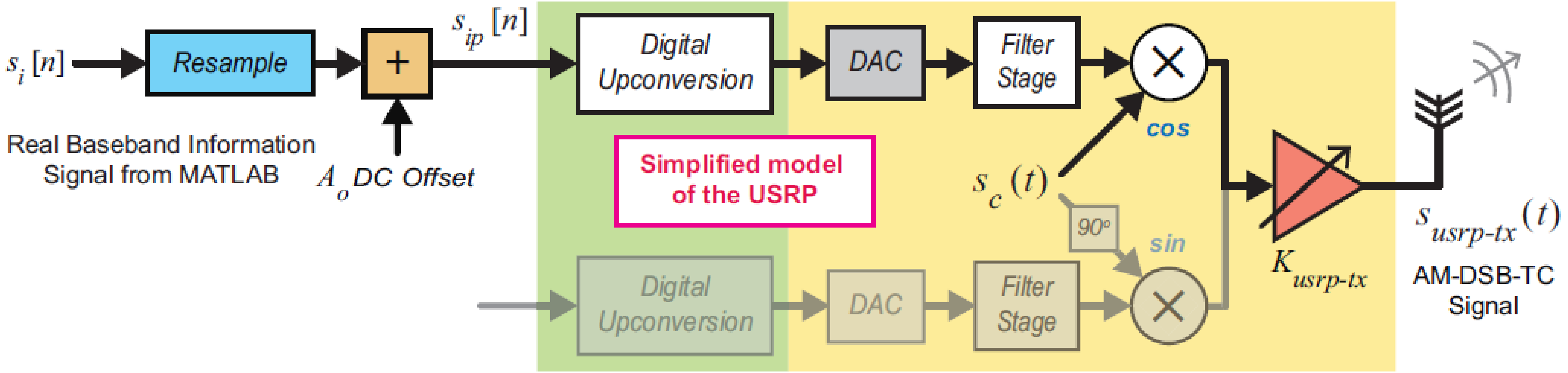


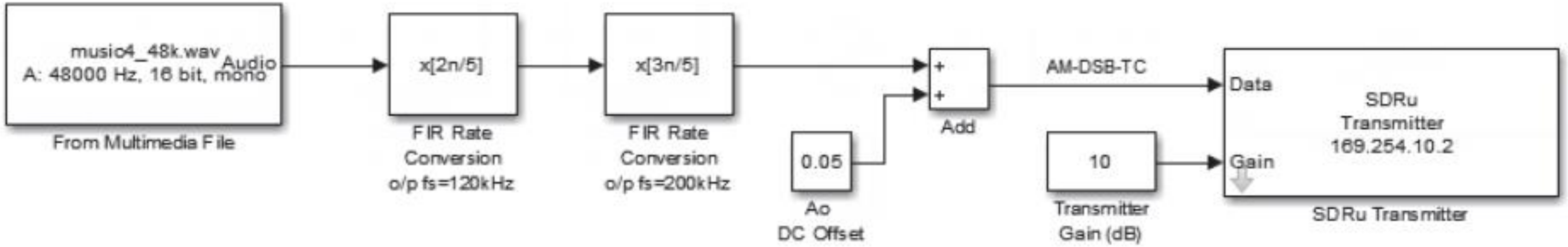




Only send real data!!!

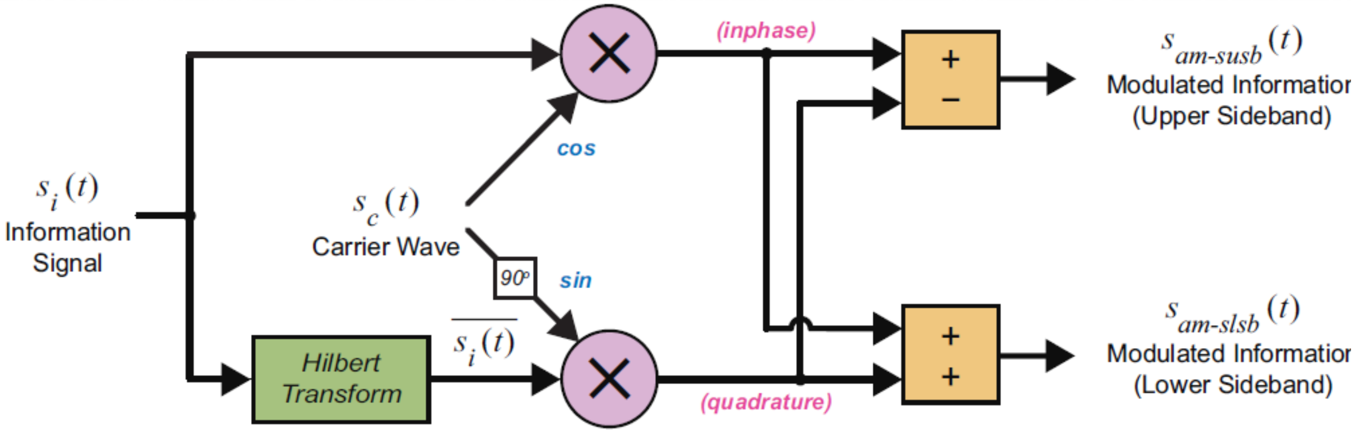
USRP TX AM DSB-TC

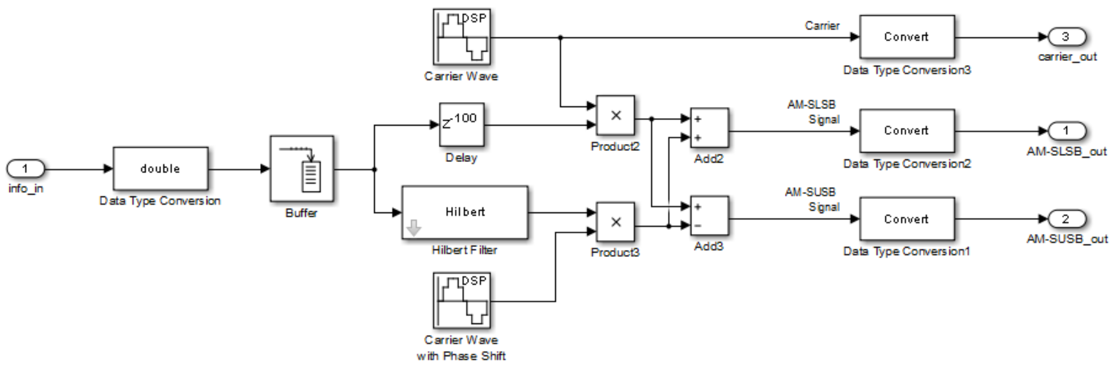


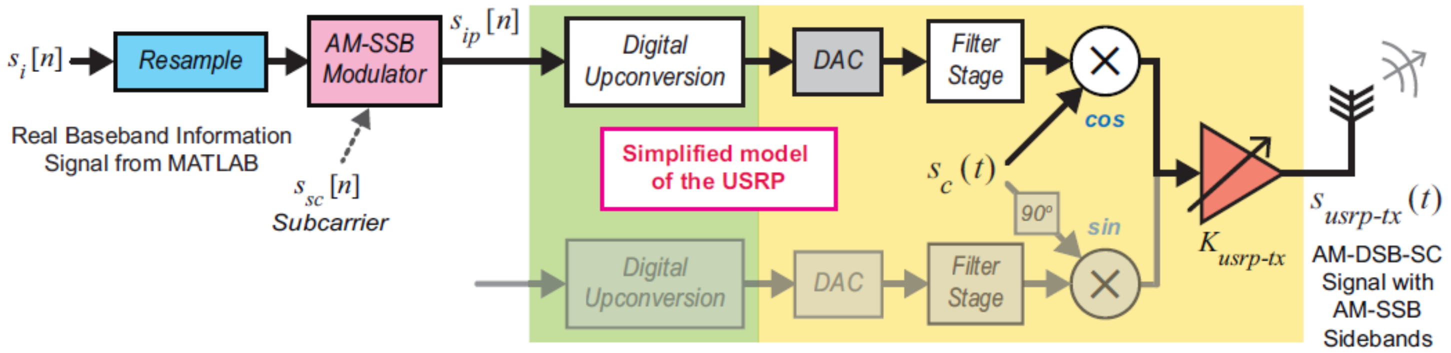


AM-SSB

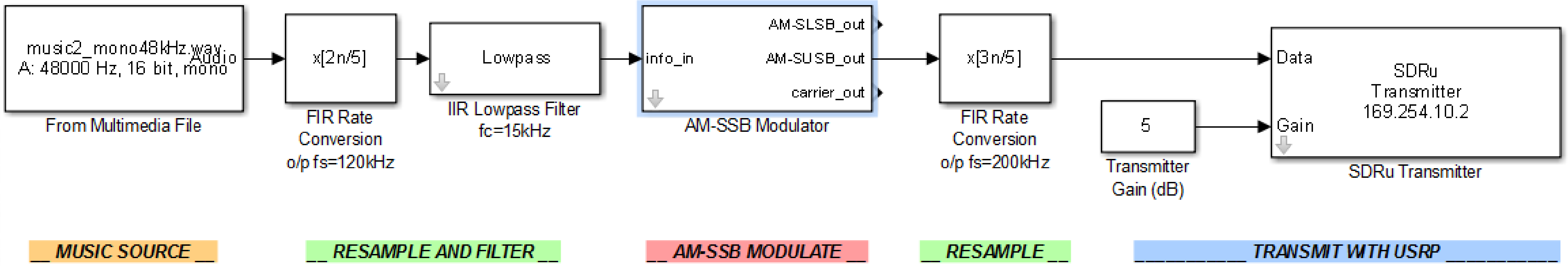
$$s_{am-ssb}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t) \mp A_i \sin(\omega_i t) A_c \sin(\omega_c t)$$

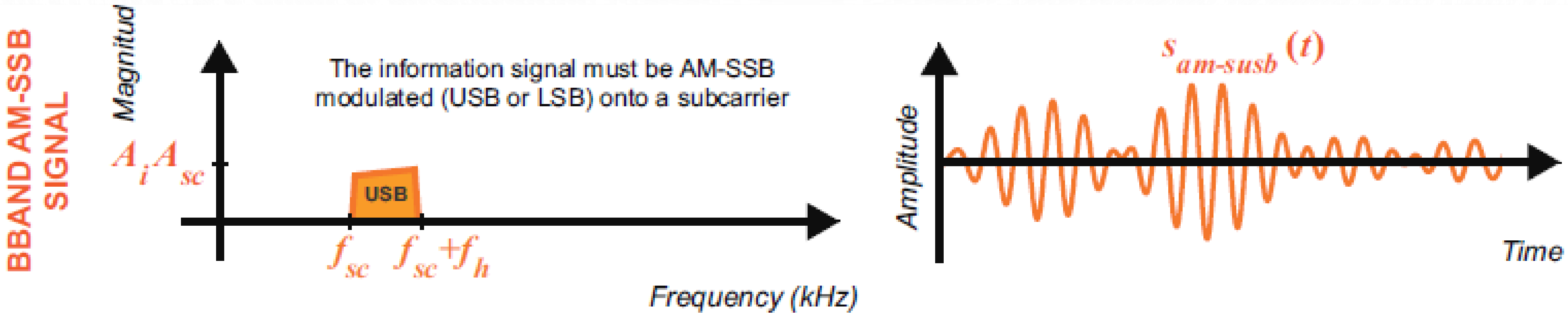






AM-SSB


music2_mono48kHz.wav
A: 48000 Hz, 16 bit, mono
Audio
From Multimedia File
x[2n/5]
FIR Rate
Conversion
o/p fs=120kHz
Lowpass
IIR Lowpass Filter
fc=15kHz
info_in
AM-SLSB_out
AM-SUSB_out
carrier_out
AM-SSB Modulator
x[3n/5]
FIR Rate
Conversion
o/p fs=200kHz
5
Transmitter
Gain (dB)
Data
Gain
SDRu
Transmitter
169.254.10.2
SDRu Transmitter
MUSIC SOURCE
RESAMPLE AND FILTER
AM-SSB MODULATE
RESAMPLE
TRANSMIT WITH USRP

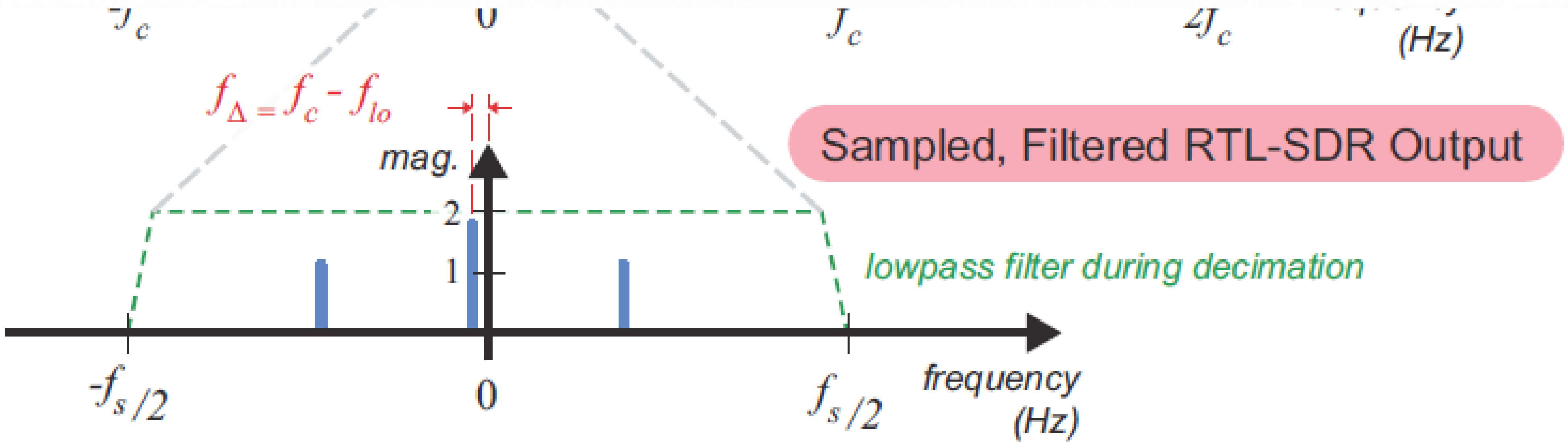

BBAND AM-SSB
SIGNAL
Magnitud
$A_i A_{sc}$
The information signal must be AM-SSB
modulated (USB or LSB) onto a subcarrier
USB
f_{sc}
$f_{sc}+f_h$
Frequency (kHz)
Amplitude
$s_{am\text{-}susb}(t)$
Time

RECEIVER DSB-TC WITH ENVELOPE DETECTOR

- Demo Ideal

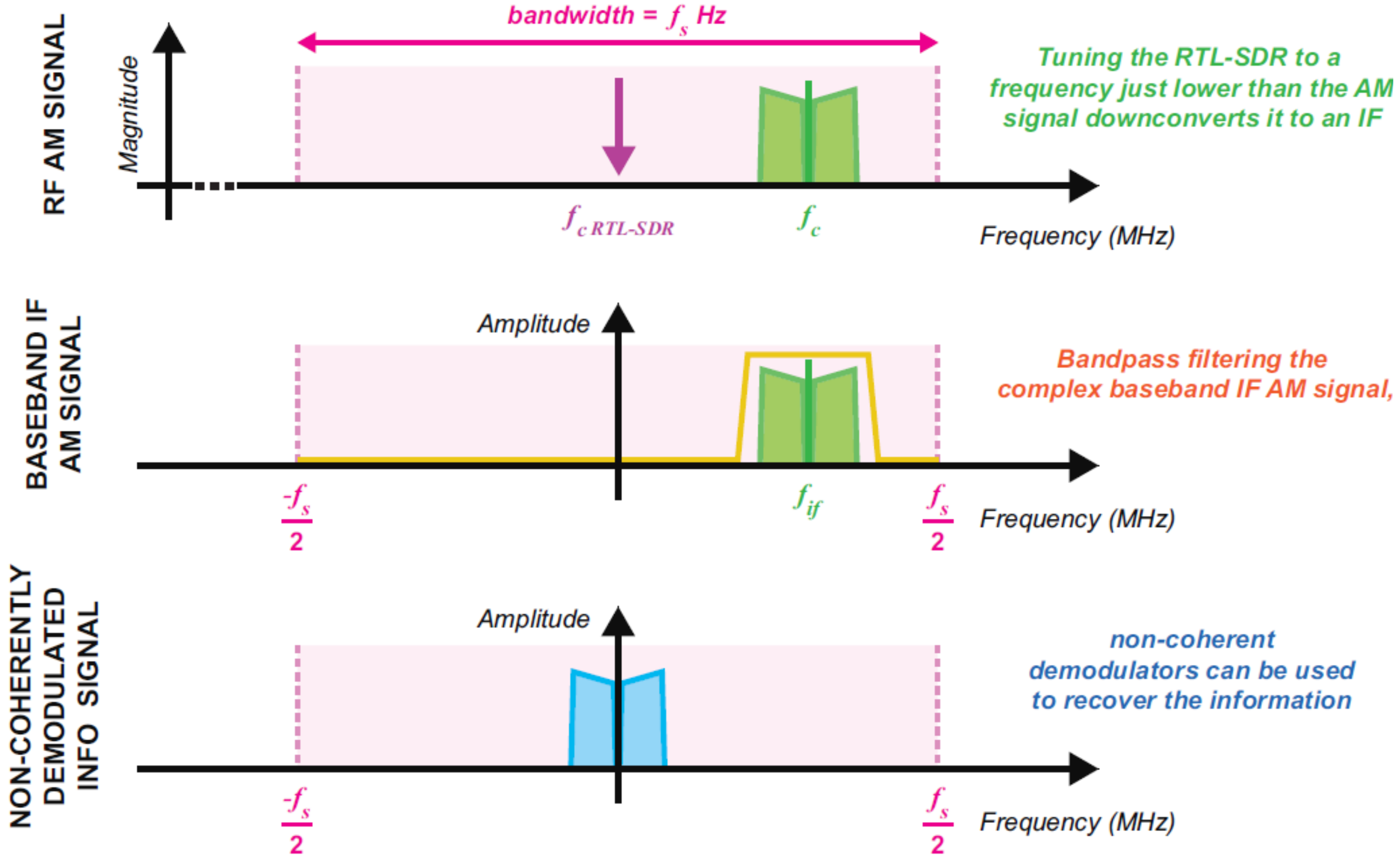
$$s_{amRF}(t) = s_{am-dsb-tc}(t) = A_o A_c \cos(\omega_c)t + \frac{A_i A_c}{2}\cos(\omega_c - \omega_i)t + \frac{A_i A_c}{2}\cos(\omega_c + \omega_i)t$$

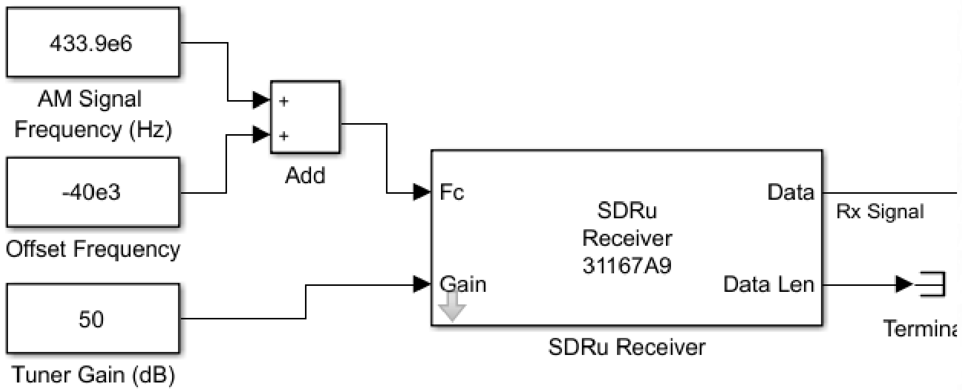
- PROBLEM: it is probable that we have frequency offset, so the AM will be modulated but a low frequency near DC. Envelop works if fc>>fi.



RX NON-COHERENT

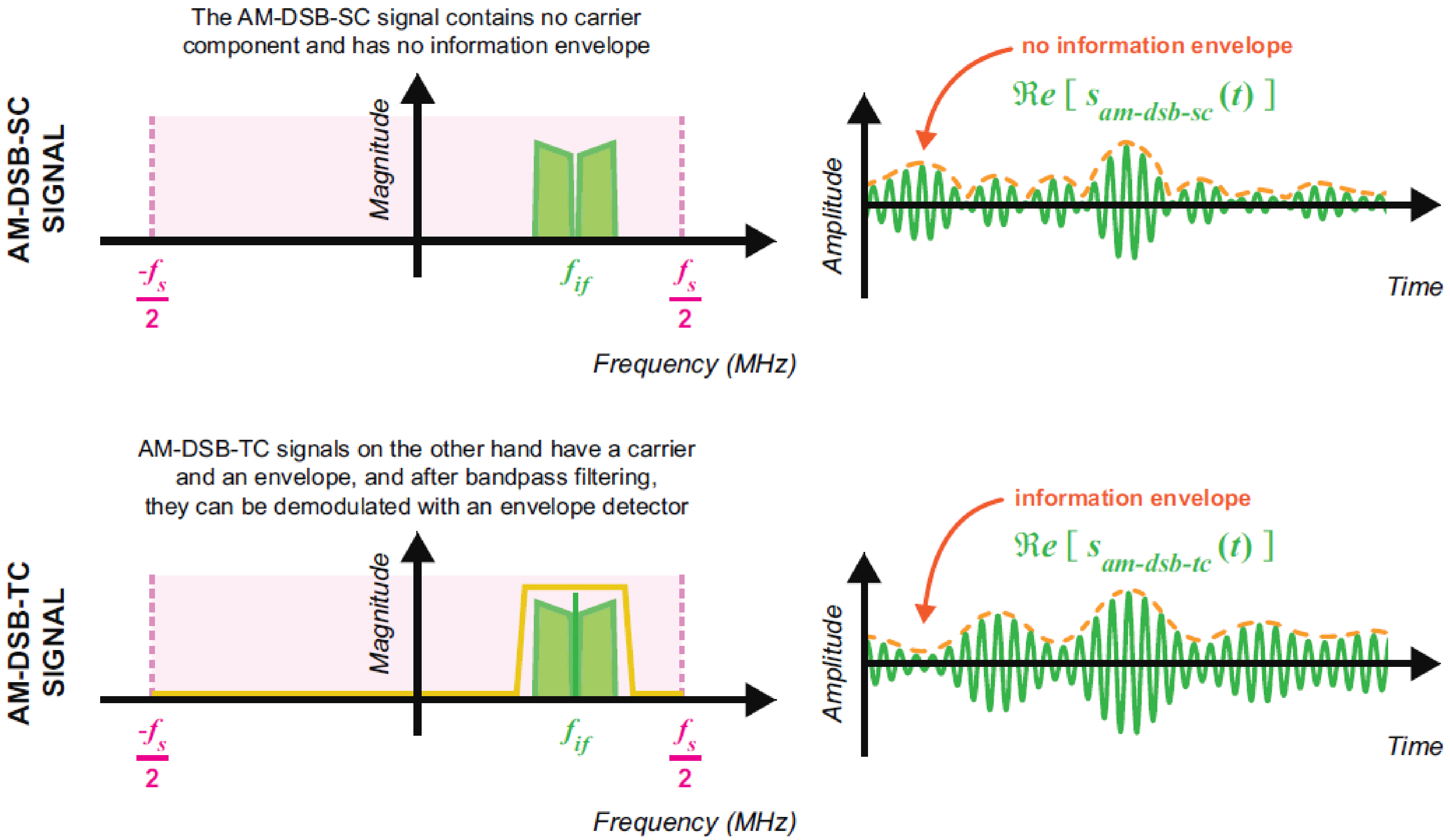
- SOLUTION: Use an offset intermediate freq. $f_{c\,(rtl-sdr\ tuner)} = f_{c\,(am\ signal)} - f_{offset}$
- Demodulated signal will be on IF area, after that envelope detector can be applied.





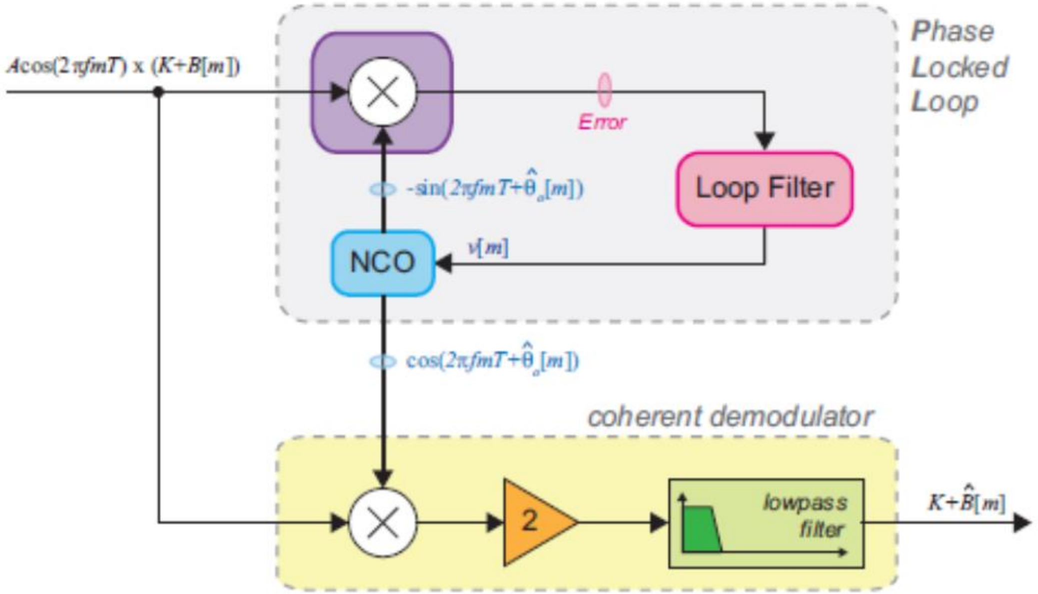
RX NON-COHERENT

- We cannot recover DSB-SC signal we envelope detector.



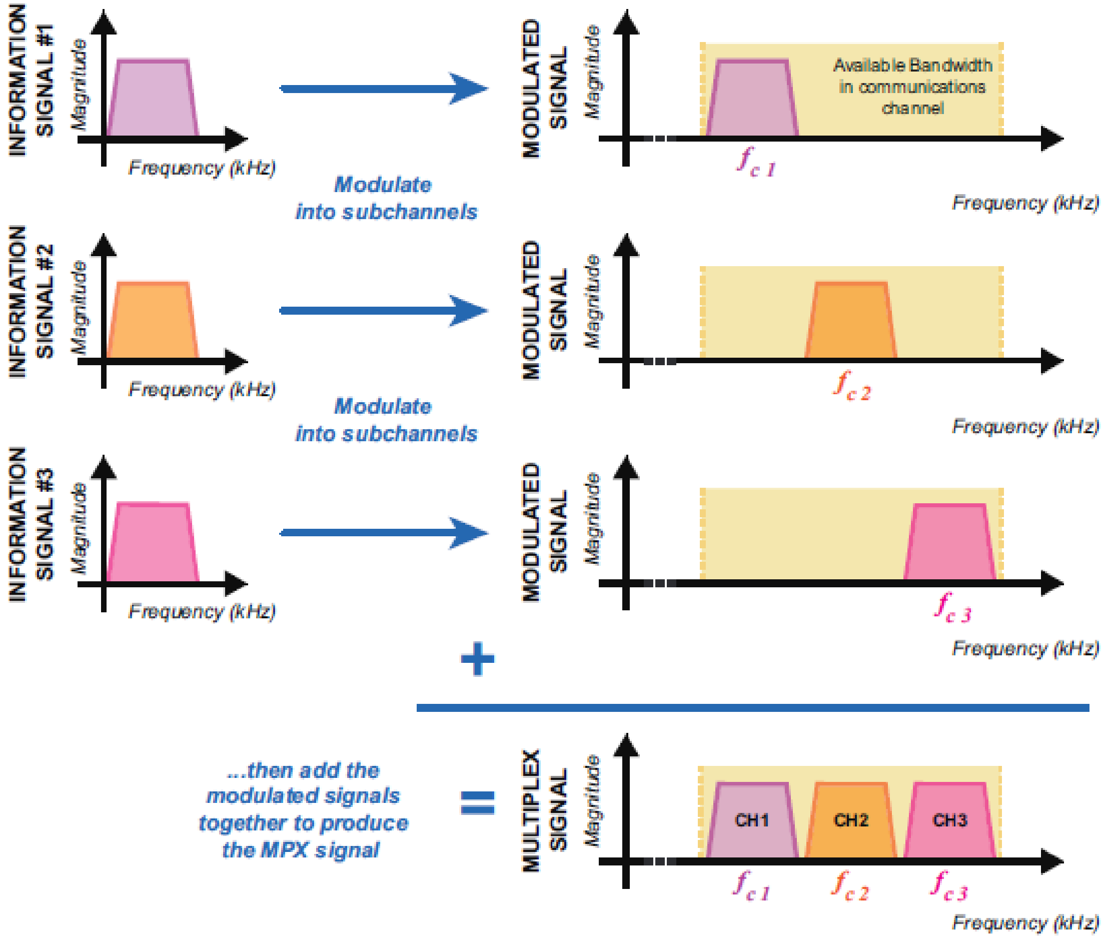
RX COHERENT

- We need a coherent receiver that use a PLL to recover the phase and frequency.



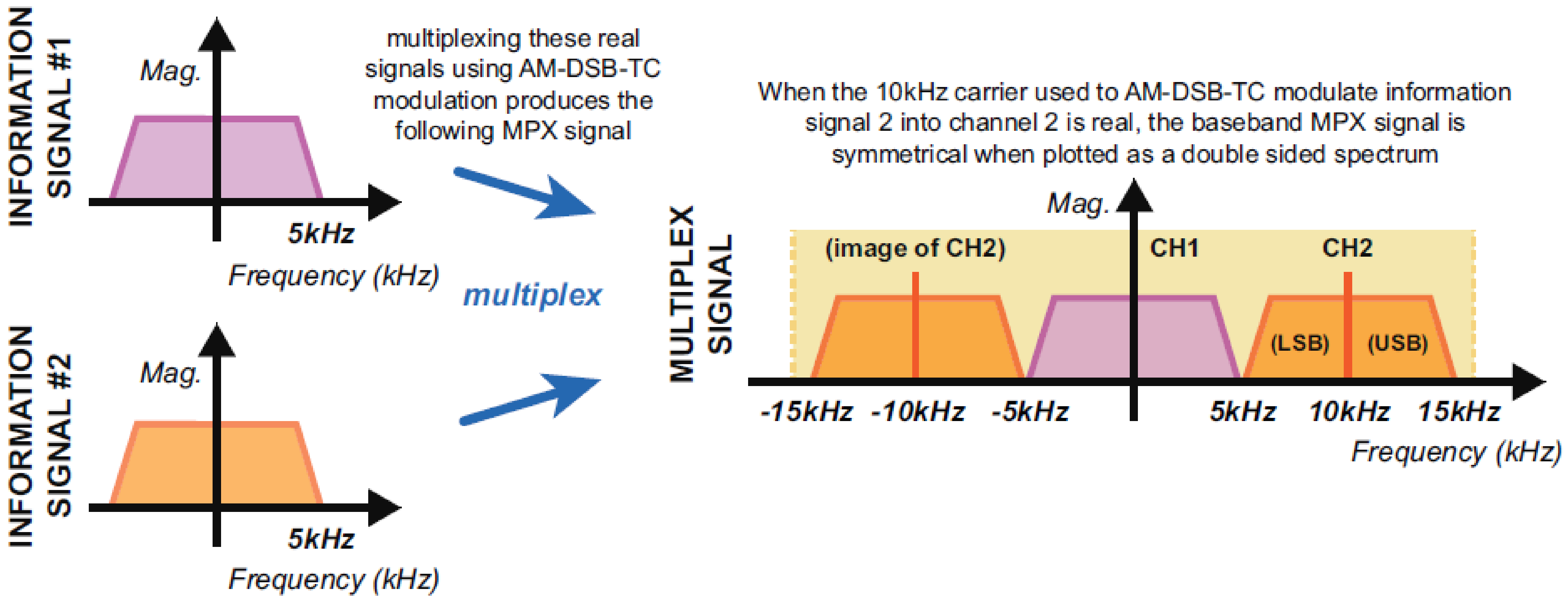
FDM INTRO

- FDM allows to send multiple signals at the same time, sharing the available spectrum.
- The first application of FDM was on wired telephony, where each user has 3.4Khz of the total bandwidth.



FDM TX

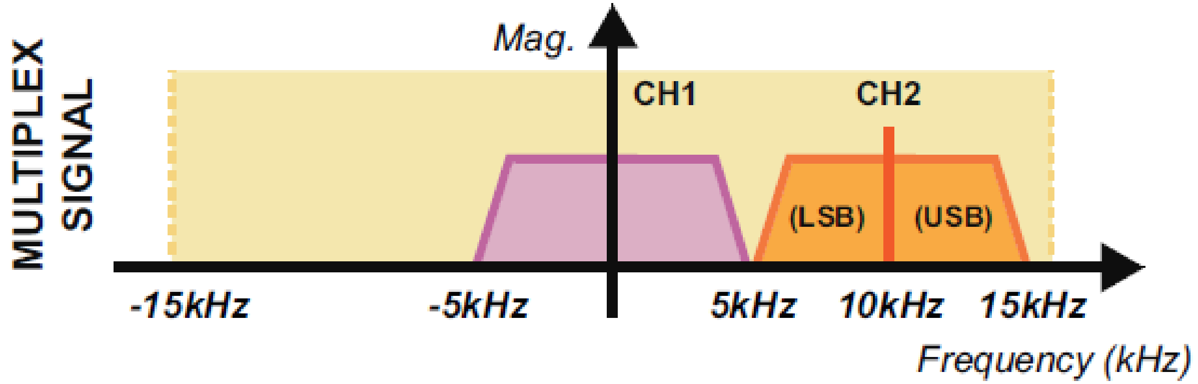
- DSB-TC modulation with real carrier



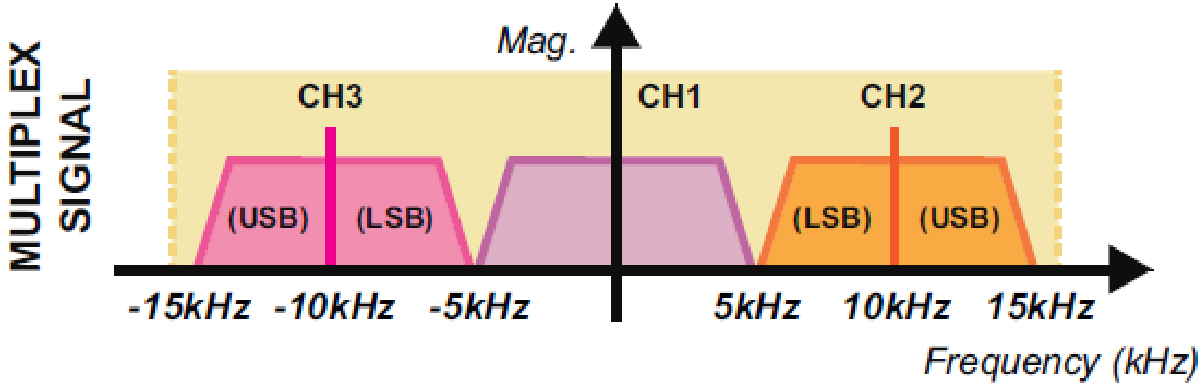
FDM TX

- Quadrature modulation, complex carrier.

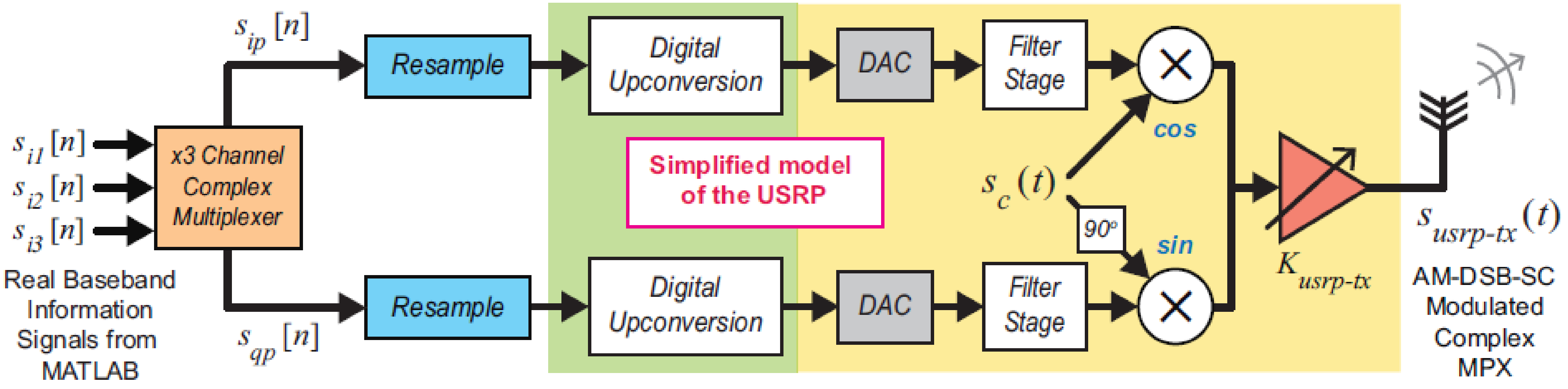
When the 10kHz carrier used to AM-DSB-TC modulate information signal 2 into channel 2 is complex, the baseband MPX signal would be asymmetrical when plotted as a double sided spectrum



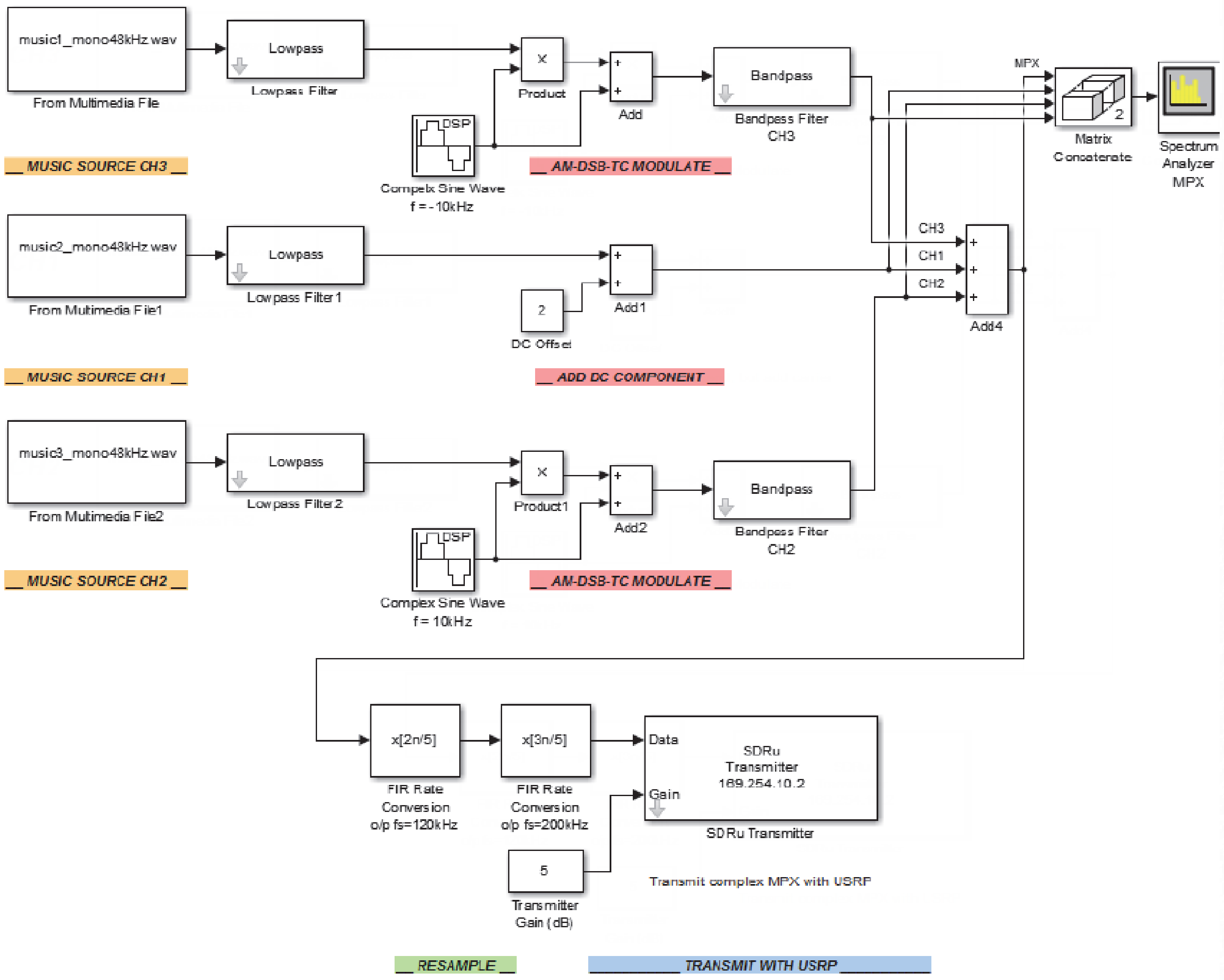
If another complex carrier (this time with a frequency of -10kHz) was used, it would be possible to position another channel in the negative part of the double sided spectrum. The overall bandwidth of the MPX signal remains the same, but another whole channel can be added by simply making the MPX complex.



FDM- TX USRP

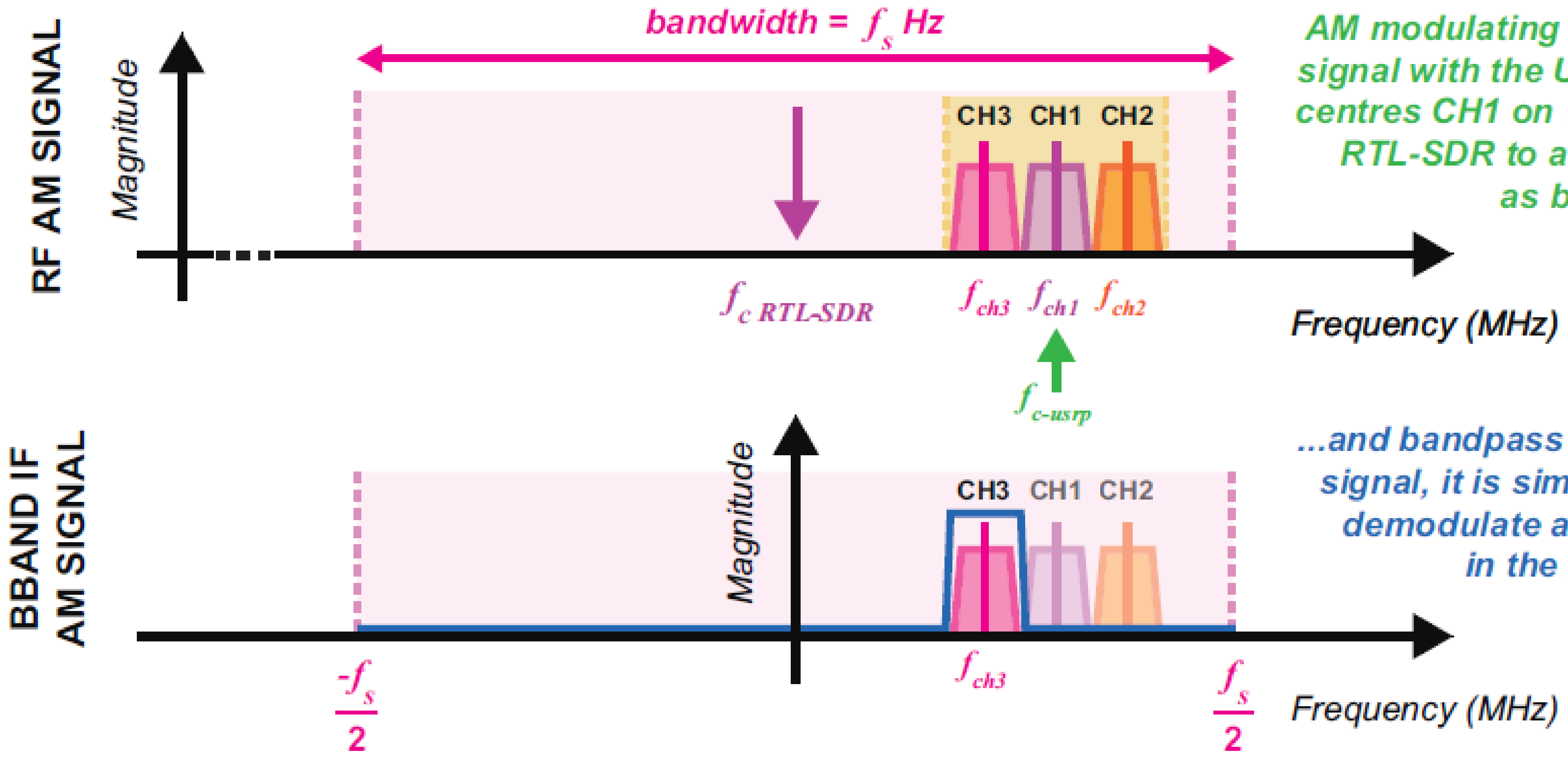

$s_{ip}[n]$
$s_{qp}[n]$
$s_{i1}[n]$
$s_{i2}[n]$
$s_{i3}[n]$
x3 Channel Complex Multiplexer
Real Baseband Information Signals from MATLAB
Resample
Resample
Digital Upconversion
Digital Upconversion
DAC
DAC
Filter Stage
Filter Stage
Simplified model of the USRP
$s_c(t)$
cos
sin
90°
$K_{usrp\text{-}tx}$
$s_{usrp\text{-}tx}(t)$
AM-DSB-SC Modulated Complex MPX

FDM TX USRP


music1_mono48kHz.wav
From Multimedia File
Lowpass
Lowpass Filter
Product
Add
Bandpass
Bandpass Filter CH3
DSP
Complex Sine Wave f = -10kHz
MUSIC SOURCE CH3
AM-DSB-TC MODULATE
MPX
Matrix Concatenate
Spectrum Analyzer MPX
music2_mono48kHz.wav
From Multimedia File1
Lowpass
Lowpass Filter1
Add1
2
DC Offset
CH3
CH1
CH2
Add4
MUSIC SOURCE CH1
ADD DC COMPONENT
music3_mono48kHz.wav
From Multimedia File2
Lowpass
Lowpass Filter2
Product1
Add2
Bandpass
Bandpass Filter CH2
DSP
Complex Sine Wave f = 10kHz
MUSIC SOURCE CH2
AM-DSB-TC MODULATE
x[2n/5]
FIR Rate Conversion o/p fs=120kHz
x[3n/5]
FIR Rate Conversion o/p fs=200kHz
Data
Gain
SDRu Transmitter 169.254.10.2
SDRu Transmitter
5
Transmitter Gain (dB)
Transmit complex MPX with USRP
RESAMPLE
TRANSMIT WITH USRP

FDM RX

Intermediate frequency receiver.



AM modulating the complex MPX signal with the USRP® transmitter centres CH1 on $f_{c\text{-}usrp}$. Tuning the RTL-SDR to a lower frequecy as before,

...and bandpass filtering the IF AM signal, it is simple to select and demodulate a single channel in the receiver