SOFTWARE DEFINED RADIO
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SESSION 2: AM MODULATION, REVIEW OF CONCEPT
REAL IMPLEMENTATION
QUADRATURE AMPLITUDE MODULATION

- AM is poor efficiency since it uses double of bandwidth.
- But is one of the simplest modulation systems, AM-DSB-SC.

\[ g(t) = A \cos(2\pi f_b t) \]
\[ c(t) = \cos(2\pi f_c t) \]
\[ s(t) = c(t) \times g(t) \]

\[ s(t) = \frac{A}{2} \cos(2\pi (f_c - f_b) t) + \frac{A}{2} \cos(2\pi (f_c + f_b) t) \]

Signal

Carrier

Modulator

Modulated Signal
BANDWIDTH EFFICIENCY

Signal $G(f)$

Carrier $\cos(2\pi f_c t)$

Modulated Signal
(bandwidth = $2f_b$ Hz)

Modulator

$T_x$

(frequency $f_b$, magnitude $2f_b$)
DSB-SC

Pros: very simple implementation
Cons: Require coherent demodulator, phase and frequency.
DSB-SC

INFORMATION SIGNAL

- Information signal is a pure cosinusoid

CARRIER WAVE

- Carrier signal is a pure cosinusoid, at a much higher frequency

AM-DSB-SC SIGNAL

- Modulated signal has two frequency components: an Upper Sideband and a Lower Sideband

Mathematical equations:

\[ s_i(t) \]

\[ s_c(t) \]

\[ s_{am-dsb-sc}(t) \]
DSB-SC BASE BAND

Consider the information signal to be a sum of three sinusoids:

\[ s_i(t) \]

\[ f_{i1} \quad f_{i2} \quad f_{i3} = f_h \]

\[ \text{bandwidth} f_h \text{ Hz} \]

AM-DSB-SC SIGNAL

\[ s_{am-dsb-sc}(t) \]

\[ f_c - f_{i3} \quad f_c - f_{i2} \quad f_c - f_{i1} \quad f_c \quad f_c + f_{i1} \quad f_c + f_{i2} \quad f_c + f_{i3} \]

\[ \text{bandwidth} 2f_h \text{ Hz} \]

amplitudes \( A \)

\[ A_{i1} / 2 \quad A_{i2} / 2 \quad A_{i3} / 2 \]
DSB-TC

\[ s_{am-dsb-tc}(t) = \left[ A_o + s_i(t) \right] \times s_c(t) \]

\[ s_{am-dsb-tc}(t) = \left[ A_o + A_i \cos(\omega_i t) \right] A_c \cos(\omega_c t) \]

\[ s_{am-dsb-tc}(t) = A_o \left[ 1 + m \cos(\omega_i t) \right] A_c \cos(\omega_c t) \]

m: modulation index
DSB-TC

Information signal is a pure cosinusoid

Carrier signal is a pure cosinusoid, at a much higher frequency

Modulated signal has three frequency components: an Upper Sideband, a Lower Sideband and a Carrier

\[ s(t) \]

\[ s_c(t) \]

\[ s_{am-dsb-tc}(t) \]
MODULATION INDEX

Pros: NOT require coherent demodulator
Cons: bad spectral efficiency
Advantages of QAM, we have better spectral efficiency.
QAM DEMODULATOR

\[ y(t) = g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \]

**Modulated Signal**

- In Phase (I): \( x_1(t) \)
- Quadrature Phase (Q): \( x_2(t) \)
- Lowpass Filter: \( z_1(t) \) and \( z_2(t) \)

**Baseband Signal**:
- Frequency: \( f_b \)
$$x_1(t) = y(t)\cos(2\pi f_c t)$$

$$= \left[ g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \right] \cos(2\pi f_c t)$$

$$= g_1(t)\cos^2(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{1}{2} g_1(t) [1 + \cos(4\pi f_c t)] - \frac{1}{2} g_2(t)\sin(4\pi f_c t)$$

$$= \frac{1}{2} g_1(t) + \left[ \frac{1}{2} g_1(t)\cos(4\pi f_c t) - \frac{1}{2} g_2(t)\sin(4\pi f_c t) \right]$$

**lowpass filtered terms**
\[ x_2(t) = y(t)(-\sin(2\pi f_c t)) \]

\[
= \left[ g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \right](-\sin(2\pi f_c t))
\]

\[
= \left[ -g_1(t)\cos(2\pi f_c t)\sin(2\pi f_c t) \right] + g_2(t)\sin^2(2\pi f_c t)
\]

\[
= -\frac{1}{2} g_1(t)\sin(4\pi f_c t) + \frac{1}{2} g_2(t)\left[ 1 - \cos(4\pi f_c t) \right]
\]

\[
= \frac{1}{2} g_2(t) - \left[ \frac{1}{2} g_1(t)\sin(4\pi f_c t) + \frac{1}{2} g_2(t)\cos(4\pi f_c t) \right]
\]

lowpass filtered terms
\[ x_1(t) = y(t) \cos(2\pi f_c t + \theta) \]

\[ = \left[ g_1(t) \cos(2\pi f_c t) - g_2(t) \sin(2\pi f_c t) \right] \cos(2\pi f_c t + \theta) \]

\[ = g_1(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \theta) - g_2(t) \sin(2\pi f_c t) \cos(2\pi f_c t + \theta) \]

\[ = \frac{1}{2}g_1(t) \left[ \cos(-\theta) + \cos(4\pi f_c t + \theta) \right] - \frac{1}{2}g_2(t) \left[ \sin(-\theta) + \sin(4\pi f_c t + \theta) \right] \]

\[ = \frac{1}{2}g_1(t) \left[ \cos(\theta) + \cos(4\pi f_c t + \theta) \right] - \frac{1}{2}g_2(t) \left[ -\sin(\theta) + \sin(4\pi f_c t + \theta) \right] \]

\[ = \frac{1}{2}g_1(t) \cos(\theta) + g_2(t) \sin(\theta) \]

\[ + \left[ \frac{1}{2}g_1(t) \cos(4\pi f_c t + \theta) - \frac{1}{2}g_2(t) \sin(4\pi f_c t + \theta) \right] \]
PHASE OFFSET

\[ z_1(t) = 0.5[g_1(t)\cos(\theta) + g_2(t)\sin(\theta)] \]

\[ z_2(t) = 0.5[-g_1(t)\sin(\theta) + g_2(t)\cos(\theta)] \]
WE HAVE TO CORRECT OFFSET BEFORE START WORKING WITH SDR
\[ g(t) = g_1(t) + jg_2(t) \]

\[ e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t) \]
\[ v(t) = g(t)e^{j2\pi f_c t} = \left[ g_1(t) + jg_2(t) \right]e^{j2\pi f_c t} \]

\[ = \left[ g_1(t) + jg_2(t) \right] \left[ \cos(2\pi f_c t) + j \sin(2\pi f_c t) \right] \]

\[ = g_1(t)\cos(2\pi f_c t) + jg_2(t)\cos(2\pi f_c t) + jg_1(t)\sin(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \]

\[ = \left[ g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \right] + j \left[ g_1(t)\sin(2\pi f_c t) + g_2(t)\cos(2\pi f_c t) \right] \]

\[ \mathcal{R}e\{v(t)\} = \mathcal{R}e\left\{ \left[ g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \right] + j \left[ g_1(t)\sin(2\pi f_c t) + g_2(t)\cos(2\pi f_c t) \right] \right\} \]

\[ = g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t) \]
REAL HARDWARE VS COMPLEX NOTATION
The simplest receiver will be a DSB-SC, because we just need to multiply by the carrier.

This will work ONLY if no phase or frequency offset.

\[ s_{am-dsb-sc}(t) = \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t \right) \]

\[ s_d(t) = \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t \right) \cos(\omega_c t) \]

\[ = \frac{A_i A_c}{2} \left[ \cos((\omega_c - \omega_i)t) \cos(\omega_c t) + \cos((\omega_c + \omega_i)t) \cos(\omega_c t) \right] \]

\[ = \frac{A_i A_c}{2} \left[ \frac{1}{2} \cos(-\omega_i t) + \frac{1}{2} \cos((2 \omega_c - \omega_i)t) \right] \]

\[ + \frac{1}{2} \cos(\omega_i t) \]

\[ + \frac{1}{2} \cos((2 \omega_c + \omega_i)t) \]

(lowpass filtered)

\[ s_d(t) = \frac{A_i A_c}{4} \cos(-\omega_i t) + \frac{A_i A_c}{4} \cos(\omega_i t) = \frac{A_i A_c}{2} \cos(\omega_i t) \]
REAL COHERENT DEMODULATOR DSB-TC

\[ s_{amRF}(t) = s_{am-dsb-tc}(t) = A_o A_c \cos(\omega_c t) + \frac{A_i A_c}{2} \cos(\omega_c - \omega_i) t + \frac{A_i A_c}{2} \cos(\omega_c + \omega_i) t \]
REAL COHERENT DEMODULATOR

\[ s_{bband}(t) = s_{amRF}(t)e^{-j\omega_{lo}t} \]

\[ = s_{amRF}(t) \times \left( \cos(\omega_{lo}t) - j\sin(\omega_{lo}t) \right) \]

\[ = A_o A_c \cos(\omega_c t) \times \left( \cos(\omega_{lo}t) - j\sin(\omega_{lo}t) \right) \]

\[ + \frac{A_i A_c}{2} \left( \cos(\omega_c t - \omega_{lo}t) + \cos(\omega_c t + \omega_{lo}t) \right) \times \left( \cos(\omega_{lo}t) - j\sin(\omega_{lo}t) \right) \]

\[ s_{bband}(t) = \frac{A_o A_c}{2} \left[ \cos(\omega_c t - \omega_{lo}t) + \cos(\omega_c t + \omega_{lo}t) \right. \]

\[ \left. - j\sin(\omega_c t - \omega_{lo}t) - j\sin(\omega_c t + \omega_{lo}t) \right] \]

\[ + \frac{A_i A_c}{4} \left[ \cos(\omega_c t - \omega_{lo}t - \omega_{lo}t) + \cos(\omega_c t - \omega_{lo}t + \omega_{lo}t) \right. \]

\[ + \cos(\omega_c t + \omega_{lo}t - \omega_{lo}t) + \cos(\omega_c t + \omega_{lo}t + \omega_{lo}t) \]

\[ - j\sin(\omega_c t - \omega_{lo}t - \omega_{lo}t) - j\sin(\omega_c t - \omega_{lo}t + \omega_{lo}t) \]

\[ - j\sin(\omega_c t + \omega_{lo}t - \omega_{lo}t) - j\sin(\omega_c t + \omega_{lo}t + \omega_{lo}t) \].
PERFECT DEMODULATOR

- Real demodulator, w/offset

\[ s_{\text{RTL-SDR}}(t) = \frac{A_o A_c}{2} \left[ \cos(\omega_c t - \omega_{lo} t) - j \sin(\omega_c t - \omega_{lo} t) \right] + \frac{A_i A_c}{4} \left[ \cos(\omega_c t - \omega_{lo} t - \omega_f t) + \cos(\omega_c t - \omega_{lo} t + \omega_f t) - j \sin(\omega_c t - \omega_{lo} t - \omega_f t) - j \sin(\omega_c t - \omega_{lo} t + \omega_f t) \right]. \]

- Perfect demo

\[ s_{\text{RTL-SDR}}(t) = \frac{A_o A_c}{2} \left[ \cos(\theta) - j \sin(\theta) \right] + \frac{A_i A_c}{4} \left[ \cos(-\omega_f t) + \cos(\omega_f t) - j \sin(-\omega_f t) - j \sin(\omega_f t) \right]. \]

\[ s_{ip}(t) = \Re \left[ s_{\text{RTL-SDR}}(t) \right] = \frac{A_o A_c}{2} + \frac{A_i A_c}{4} \left[ \cos(-\omega_f t) + \cos(\omega_f t) \right] \quad s_{iq}(t) = \Im \left[ s_{\text{RTL-SDR}}(t) \right] = -j \frac{A_i A_c}{4} \left[ \sin(-\omega_f t) + \sin(\omega_f t) \right]. \]

- Because is hard to get “perfect tuning” on the carrier, we need a coherent receiver with frequency and phase recovery to avoid this problems.
DEMODULATED RECEIVED SPECTRUM

A. Received AM Signal

B. Complex Baseband AM

C. Sampled, Filtered RTL-SDR Output
EASY way is to filter AM-DSB-SC, BUT filter needs very large attenuation and rolloff factor.

\[ s_{am-ssb}(t) = s_i(t) \Re \left[ s_c(t) \right] + s_i(t) \Im \left[ s_c(t) \right] \]
SSB TRANSMITTER

\[ s_i(t) \rightarrow \overline{s_i(t)} \]

\[ A_i \cos(\omega_i t) \rightarrow A_i \sin(\omega_i t) \]

\[ s_c(t) = A_c \cos(2\pi f_c t) + A_c \sin(2\pi f_c t) \]

\[ s_{am-ssb}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t) \mp A_i \sin(\omega_i t) A_c \sin(\omega_c t) \]

\[ s_{am-ssb}(t) = \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i) t + \cos(\omega_c + \omega_i) t \right) \]

\[ - \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i) t - \cos(\omega_c + \omega_i) t \right) \]

\[ = A_i A_c \cos(\omega_c + \omega_i) t \]

\[ s_{am-ssb}(t) = \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i) t + \cos(\omega_c + \omega_i) t \right) \]

\[ + \frac{A_i A_c}{2} \left( \cos(\omega_c - \omega_i) t - \cos(\omega_c + \omega_i) t \right) \]

\[ - A_i A_c \cos(\omega_c - \omega_i) t . \]
• It was created to reduce the bandwidth on analog TV channels that use DSB-TC.

• The modulator is DSB-TC, but with a pass-band filter at the output. This filter allows to pass the carrier plus a portion of the lower band, the vestigial band.
• Transmitter
NON-COHERENT DEMO: ENVELOPE DETECTOR

- Classical detector, only works for DSB-TC
• USRP hardware support package is required.
• By default only digital up-conversion is done on the FPGA
**USRP MATLAB PARAMETERS**

- **TX Gain**: expressed as scalar. Max 90dB
- **RX Gain**: 73dB
- **Clock Freq**: 5Mhz a 56Mhz
- **DUC**: default 512, correspond to simple rate of $10^8/512=195$Khz
- **Underrun**: “+” not enough samples, “0” no data loss
- **Enable burst**: no underrun / overrun, fixed number of frames
AM TRANSMISSION

USRP TX AM DSB-SC

Only send real data!!!
AM-SSB

\[ s_{am-ssb}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t) \mp A_i \sin(\omega_i t) A_c \sin(\omega_c t) \]

\[ s_i(t) \] Information Signal
\[ s_c(t) \] Carrier Wave
\[ \cos \] Hilbert Transform
\[ \sin \] Resample

\[ s_{am-ssb}(t) \] Modulated Information (Upper Sideband)
\[ s_{am-slb}(t) \] Modulated Information (Lower Sideband)

Real Baseband Information Signal from MATLAB
\[ s_{am}[n] \] Subcarrier

Simplified model of the USRP

AM-DSB-SC Signal with AM-SSB Sidebands
AM TRANSMISSION

AM-SSB

The information signal must be AM-SSB modulated (USB or LSB) onto a subcarrier.
• Demo Ideal

\[ s_{amRF}(t) = s_{am-dsb-tc}(t) = A_o A_c \cos(\omega_c)t + \frac{A_i A_c}{2} \cos(\omega_c - \omega_i)t + \frac{A_i A_c}{2} \cos(\omega_c + \omega_i)t \]

• PROBLEM: it is probable that we have frequency offset, so the AM will be modulated but a low frequency near DC. Envelop works if \( fc >> fi \).
AM RECEIVER

- SOLUTION: Use an offset intermediate freq. \( f_c(\text{rtl-sdr tuner}) = f_c(\text{am signal}) - f_{\text{offset}} \)
- Demodulated signal will be on IF area, after that envelope detector can be applied.

**RX NON-COHERENT**

- Tuning the RTL-SDR to a frequency just lower than the AM signal downconverts it to an IF
- Bandpass filtering the complex baseband IF AM signal
- Non-coherent demodulators can be used to recover the information
• We cannot recover DSB-SC signal we envelope detector.
• We need a coherent receiver that use a PLL to recover the phase and frequency.
• FDM allows to send multiple signals at the same time, sharing the available spectrum.

• The first application of FDM was on wired telephony, where each user has 3.4Khz of the total bandwidth.
• DSB-TC modulation with real carrier

When the 10kHz carrier used to AM-DSB-TC modulate information signal 2 into channel 2 is real, the baseband MPX signal is symmetrical when plotted as a double sided spectrum.
• Quadrature modulation, complex carrier.
FDM- TX USRP

- Resample
- Digital Upconversion
- DAC
- Filter Stage
- Simplified model of the USRP
- $s_c(t)$
- $s_{\text{usrp-tx}}(t)$

$x3$ Channel Complex Multiplexer
Real Baseband Information Signals from MATLAB

- $s_{ip}[n]$
- $s_{iq}[n]$
- $s_{iq}[n]$
- $s_{q_p}[n]$
Intermediate frequency receiver.

AM modulating the complex MPX signal with the USRP® transmitter centres CH1 on \( f_{c\text{-usrp}} \). Tuning the RTL-SDR to a lower frequency as before,

...and bandpass filtering the IF AM signal, it is simple to select and demodulate a single channel in the receiver.