SOFTWARE DEFINED RADIO
USR SDR WORKSHOP, SEPTEMBER 2017
PROF. MARCELO SEGURA

SESSION 3: FREQUENCY MODULATION
The simplest FM modulator is a VCO, frequency changes depend on amplitude input. The equation for the angle change is:

\[ \dot{\theta}(t) = k_o \int_{-\infty}^{t} v(t) \, dt \]

The modulated carrier can be expressed as:

\[ c(t) = A_o \cos \left( 2\pi f_c t + \dot{\theta}(t) \right) \]
• Considering an information signal

\[ s_i(t) = A_i \cos(2\pi f_i t) = A_i \cos(\omega_i t) \]

\[ \theta_{fm}(t) = 2\pi K_{fm} A_i \times \int_{-\infty}^{t} \cos(\omega_i t) \, dt \]

\[ = 2\pi K_{fm} A_i \times \frac{\sin(\omega_i t)}{\omega_i} \]

\[ = \frac{K_{fm} A_i}{f_i} \sin(\omega_i t) \]

• Frequency Deviation

\[ \Delta f \frac{f_i}{f} \sin(\omega_i t) \]

• Modulation index

\[ s_{fm}(t) = A_c \cos(\omega_c t + \beta_{fm} \sin(\omega_i t)) \]
NARROW FM

• If modulation index $\ll 1$ NFM or $\gg 1$ WFM

$$s_{fm}(t) = A_c \cos \left( \omega_c t + \beta_{fm} \sin(\omega_i t) \right) = A_c \cos(\omega_c t) \cos \left( \beta_{fm} \sin(\omega_i t) \right) - A_c \sin(\omega_c t) \sin \left( \beta_{fm} \sin(\omega_i t) \right)$$

$$\cos \left( \beta_{fm} \sin(\omega_i t) \right) \approx 1 \quad \text{and} \quad \sin \left( \beta_{fm} \sin(\omega_i t) \right) \approx \beta_{fm} \sin(\omega_i t)$$

• Replacing back it will look similar to AM-DSB-TC

$$s_{fm-nfm}(t) = A_c \cos(\omega_c t) - A_c \sin(\omega_c t) \beta_{fm} \sin(\omega_i t)$$

$$= A_c \left[ \cos(\omega_c t) + \frac{\beta_{fm}}{2} \cos(\omega_c + \omega_i) t - \frac{\beta_{fm}}{2} \cos(\omega_c - \omega_i) t \right]$$
WIDEBAND FM

- WFM is the standard used by commercial radio stations and it has a frequency deviation of 75kHz and a limited bandwidth of 200Khz.

- On WFM we have to solve:

  \[
  \cos\left(\beta_{fm} \sin(\omega_i t)\right) \quad \text{and} \quad \sin\left(\beta_{fm} \sin(\omega_i t)\right)
  \]

- Using the Bessel we get:

  \[
  \cos\left(\beta_{fm} \sin(\omega_i t)\right) = J_0(\beta_{fm}) + 2 \sum_{n=1}^{\infty} J_{2n}(\beta_{fm}) \cos(2n\omega_i t)
  \]

  \[
  = J_0(\beta_{fm}) + 2 J_2(\beta_{fm}) \cos(2\omega_i t) + 2 J_4(\beta_{fm}) \cos(4\omega_i t) + \ldots
  \]

  \[
  \equiv J_0 + 2 J_2 \cos(2\omega_i t) + 2 J_4 \cos(4\omega_i t) + \ldots
  \]
If we consider just a tone to be transmitted:

\[ s_{fm-wfm}(t) = A_c J_0 \cos(\omega_c t) \]

\[-A_c J_1 \left[ \cos(\omega_c - \omega_i) t - \cos(\omega_c + \omega_i) t \right] \]

\[+ A_c J_2 \left[ \cos(\omega_c - 2\omega_i) t + \cos(\omega_c + 2\omega_i) t \right] \]

\[-A_c J_3 \left[ \cos(\omega_c - 3\omega_i) t - \cos(\omega_c + 3\omega_i) t \right] \]

\[+ A_c J_4 \left[ \cos(\omega_c - 4\omega_i) t + \cos(\omega_c + 4\omega_i) t \right] + \ldots \]

The WFM signal has an infinite number of frequency components, and every odd numbered LSB is 180 degrees out of phase with the others.
The bandwidth of a WFM signal can be estimated by finding the frequencies of the highest and lowest sideband components that contain a significant amount of power.

Usually, we don’t know \( n \) so it is estimated:

\[
B = 2nf_i \text{ Hz}
\]

The BW is estimated using the Carlson Rule

\[
B = 2\left( \beta_{fm} + 1 \right)f_i
\]

\[
= 2\left( \frac{\Delta f}{f_i} + 1 \right)f_i
\]

\[
= 2\left( \Delta f + f_i \right) \text{ Hz}
\]
Differentiator Demodulator

\[ s_{fm}(t) = A_c \cos\left( \omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt \right) \]

\[ s_{fm}'(t) = \frac{d}{dt} s_{fm}(t) = -A_c \left[ \omega_c + 2\pi K_{fm} s_i(t) \right] \sin\left( \omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt \right). \]

The fluctuations in this envelope are directly proportional to the instantaneous frequency.
RX COMPLEX BASEBAND

• The SDR has a quadrature downconverter. The baseband output signal look like:

\[ s_{bband}(t) = s_{\text{fmRF}}(t)e^{-j\omega_{c}t} = s_{\text{fmRF}}(t) \left( \cos(\omega_{c}t) - j\sin(\omega_{c}t) \right) = A_{c} \cos(\omega_{c}t + \theta_{\text{fm}}(t)) \left( \cos(\omega_{lo}t) - j\sin(\omega_{lo}t) \right) \]

\[ = \frac{A_{c}}{2} \left[ \cos(\omega_{c}t + \theta_{\text{fm}}(t) - \omega_{lo}t) + \cos(\omega_{c}t + \theta_{\text{fm}}(t) + \omega_{lo}t) - j\frac{A_{c}}{2} \left[ \sin(\omega_{c}t + \theta_{\text{fm}}(t) + \omega_{lo}t) - \sin(\omega_{c}t + \theta_{\text{fm}}(t) - \omega_{lo}t) \right] \right] \]

• High frequency component removed by internal low pass filter.

• Close to zero \( \omega_{\Delta} = \omega_{c} - \omega_{lo} \)

\[ s_{bband}(t) = s_{\text{fmRF}}(t)e^{-j\omega_{c}t} = \frac{A_{c}}{2} \left[ \cos(\omega_{\Delta}t + \theta_{\text{fm}}(t)) + j\sin(\omega_{\Delta}t + \theta_{\text{fm}}(t)) \right] = \frac{A_{c}}{2} e^{j(\omega_{\Delta}t + 2\pi K_{\text{fm}} \times \int_{-\infty}^{t} s_{x}(t)dt)} \]
COMPLEX BASEBAND RX

Received FM Signal

Complex Baseband FM

Sampled, Filtered RTL-SDR Output

lowpass filter during decimation
DEMO 1: COMPLEX DIFFERENTIATOR

• Differentiate both branch I/Q

\[
\frac{A_c}{2} \left[ \cos(\omega_\Delta t + \theta_{fm}(t)) + j \sin(\omega_\Delta t + \theta_{fm}(t)) \right]
\]

\[
s_{ip}'(t) = \frac{d}{dt} s_{ip}(t) = -\frac{A_c}{2} \left[ \omega_\Delta + \theta_{fm}'(t) \right] \sin(\omega_\Delta t + \theta_{fm}(t))
\]

\[
s_{qp}'(t) = \frac{d}{dt} s_{qp}(t) = \frac{A_c}{2} \left[ \omega_\Delta + \theta_{fm}'(t) \right] \cos(\omega_\Delta t + \theta_{fm}(t))
\]

• Mixed them

\[
s_{ip}'(t) \times s_{qp}(t) = -\frac{A_c^2}{4} \left[ \omega_\Delta + \theta_{fm}'(t) \right] \sin^2(\omega_\Delta t + \theta_{fm}(t))
\]

\[
s_{qp}'(t) \times s_{ip}(t) = \frac{A_c^2}{4} \left[ \omega_\Delta + \theta_{fm}'(t) \right] \cos^2(\omega_\Delta t + \theta_{fm}(t))
\]

• Subtract the terms

\[
s_\alpha(t) = (s_{qp}'(t) \times s_{ip}(t)) - (s_{ip}'(t) \times s_{qp}(t)) = \frac{A_c^2}{4} \left[ \omega_\Delta + \theta_{fm}'(t) \right]
\]
DEMO 1: COMPLEX DIFFERENTIATOR

\[
\theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t) dt
\]

\[
s_n(t) = \frac{s_{\alpha}(t)}{s_p(t)} = \frac{A_c^2}{4} \left[ \omega_\Delta + \theta_{fm}'(t) \right]
\]

\[
= \left[ \omega_\Delta + \theta_{fm}'(t) \right]
\]
**DEMO 2: COMPLEX DELAY LINE**

Phase detection

\[
s_{\text{delay}}(t) = \frac{A_c}{2} e^{j(\omega_{\Delta}[t - \tau] + \theta_{fm}(t - \tau))}
\]

\[
s_{\text{pd}}(t) = s_{\text{conj}}(t) \times s_{\text{delay}}(t)
\]

\[
s_{\text{conj}}(t) = \frac{A_c}{2} e^{-j(\omega_{\Delta}t + \theta_{fm}(t))}
\]

**Phase Shifter**

Complex FM Signal Received by RTL-SDR

\[
s_{\text{RTL-SDR}}[n]
\]

\[
s_{pd}[n]
\]

\[
s_{\text{conj}}[n]
\]

\[
s_{\text{delay}}[n]
\]

\[
s_{\text{sp}}[n]
\]

**Demodulated Output**

\[
s_{d}[n]
\]
**DEMO 2: COMPLEX DELAY LINE**

- Taking the angle of the signal

\[
s_d(t) = \angle s_{pd}(t) = -\left[ (\omega \Delta t + \theta_{fm}(t)) - (\omega \Delta [t - \tau] + \theta_{fm}(t - \tau)) \right]
\]

\[
= -\left[ (\omega \Delta t - \omega \Delta [t - \tau]) + (\theta_{fm}(t) - \theta_{fm}(t - \tau)) \right]
\]

- If the delay is small

\[
s_d(t) \approx -\left[ \frac{d}{dt}(\omega \Delta t) + \frac{d}{dt}(\theta_{fm}(t)) \right]
\]

\[
s_d(t) = -\left[ \omega \Delta + \theta_{fm}'(t) \right]
\]

\[
= -\left[ \omega \Delta + 2\pi K_{fm} s_i(t) \right]
\]

- Similar result as the complex differentiator, but simpler implementation.

- If you want to implement on FPGA, you have to use CORDIC algorithms to make efficient.
DEMO 3: PLL COHERENT RECEIVER

- PLL will try to follow the frequency changes, it will never lock.
- To function as an FM demodulator, the internal parameters of the PLL must be chosen appropriately.
COMMERCIAL RADIOS

• Pre-emphasis & De-Emphasis, it is done to give gain to high frequency components in order to maintain the station bandwidth.

• For US is a filter with a time constant of 75us and for Europe 50us.

• It is implemented with an analog filter.
COMMERCIAL RADIOS

- **MULTIPLEX**: today stations still transmit MONO signals for all receivers, and STEREO + RDS signal for new receivers.

- **MONO** = L+R

- **Stereo** is modulated as AM-DSB-SC a 38Khz

- The pilot inform is there is STEREO information, and is used to extract carrier information for coherent demo.

- **RDS** = Radio Data System, used to transmit the song track, station info.
COMMERCIAL RADIOS

- MULTIPLEXER TX

\[ s_{rds}(t) \]
BPSK Modulated RDS Bitstream

\[ s_l(t) \]
Left Audio Channel

\[ s_r(t) \]
Right Audio Channel

\[ s_{rds}(t) \]

\[ s_l(t) \]

\[ s_r(t) \]

\[ s_{rds}(t) \times RDS@57kHz \]

\[ [L+R]@Bband \]

\[ 57kHz \]
Freq Tripler

\[ 19kHz \]
Freq Doubler

\[ 38kHz \]

\[ [L-R] \]

\[ [L-R]@38kHz \]

\[ s_{fm\,mpx}(t) \]
FM MPX Information Signal (to modulator)
COMMERCIAL RADIOS

• DE-MULTIPLEX on RX
FM BASEBAND REPRESENTATION

\[ s_{fm}(t) = A_c \cos(\omega_c t + \theta_{fm}(t)) \quad \text{where} \quad \theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t) dt \]

\[ s_{fm-baseband}(t) = A_c e^{-j\theta_{fm}(t)} = A_c \angle -\theta_{fm}(t) \]

\[ s_{usrp-tx}(t) = K_{usrp-tx} \left[ \Re \left( s_{fm-baseband}(t) \right) \cos(\omega_c t) + \Im \left( s_{fm-baseband}(t) \right) \sin(\omega_c t) \right] \]

\[ = K_{usrp-tx} \left[ A_c \cos(-\theta_{fm}(t)) \cos(\omega_c t) + A_c \sin(-\theta_{fm}(t)) \sin(\omega_c t) \right] \]

\[ = A_c K_{usrp-tx} \cos(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t) dt) \]
\[ s_{\text{fm-baseband}}[n] = A_c K_{\text{usrp-tx}} \cos\left(\omega_c t + 2\pi K_{\text{fm}} \times \int_{-\infty}^{t} s_i(t) dt\right) \]

**FM MONO MODULATOR**

- Information Signal: \( s_i[n] \)
- Integrator
- \(-2\pi K_{\text{fm}}\)
- Mag/Arg
- DC Carrier Modulated by Information Signal

**Diagram:**

1. **Resample**
2. **Baseband FM Mod**
3. **Digital Upconversion**
4. **DAC**
5. **Filter Stage**
6. **Simplified model of the USRP**
7. \( s_{\text{ip}}[n] \)
8. \( s_{\text{qp}}[n] \)
9. \( s_c(t) \)
10. \( K_{\text{usrp-tx}} \)
11. **WFM Signal**
FM STEREO MODULATOR

$s_l[n]$
Left Audio Channel

$s_r[n]$
Right Audio Channel

$s_{mpx}[n]$
FM MPX Information Signal (to modulator)

$[L+R] @ Baseband$

Freq Doubler

$19kHz$

$38kHz$

$[L-R] @ 38kHz$

Resample

Encoder/MPX'er

Baseband FM Mod

Digital Upconversion

DAC

Filter Stage

$K_{usrp-tx}$

$s_{usrp-tx}(t)$

WFM Signal

Digital Upconversion

$s_{ip}[n]$

$s_{op}[n]$
Stereo Baseband Information Signal from MATLAB
FM STEREO DEMODULATOR

![Diagram of FM STEREO DEMODULATOR](image-url)
REAL FM MPX SIGNAL RECEIVED WITH THE RTL-SDR

RX SIGNAL

Magnitude

Mono [L+R]

Pilot

Stereo [L-R]

( Stereo LSB )

( Stereo USB )

Frequency (kHz)

15 19 23 38 53