USC Viterbi School of Engineering
Center for Cyber-Physical Systems and the Internet of Things

SOFTWARE DEFINED RADIO

USR SDR WORKSHOP, SEPTEMBER 2017

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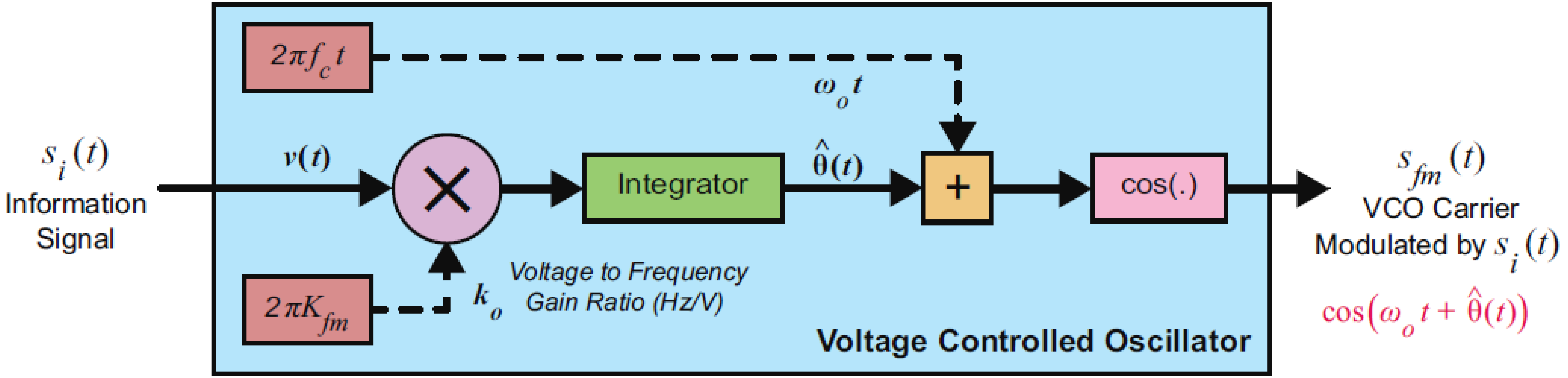
SESSION 3: FREQUENCY MODULATION

MODULATION REVIEW

The simplest FM modulator is a VCO, frequency changes depends on amplitude input



$$\hat{\theta}(t) = k_o \int_{-\infty}^{t} v(t)\,dt \qquad \hat{\theta}(t) = \theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\,dt \qquad c(t) = A_o \cos\left(2\pi f_o t + \hat{\theta}(t)\right)$$

MODULATION REVIEW

- Considering an information signal $s_i(t) = A_i \cos(2\pi f_i t) = A_i \cos(\omega_i t)$

$$\theta_{fm}(t) = 2\pi K_{fm} A_i \times \int_{-\infty}^{t} \cos(\omega_i t)\, dt$$

$$= 2\pi K_{fm} A_i \times \frac{\sin(\omega_i t)}{\omega_i}$$

$$= \frac{K_{fm} A_i}{f_i} \sin(\omega_i t)$$

- Frequency Deviation

$$= \frac{\Delta f}{f_i} \sin(\omega_i t)$$

- Modulation index

$$= \beta_{fm} \sin(\omega_i t)$$

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t) \right)$$

NARROW FM

- If modulation index **<<1 NFM** or >>1 WFM

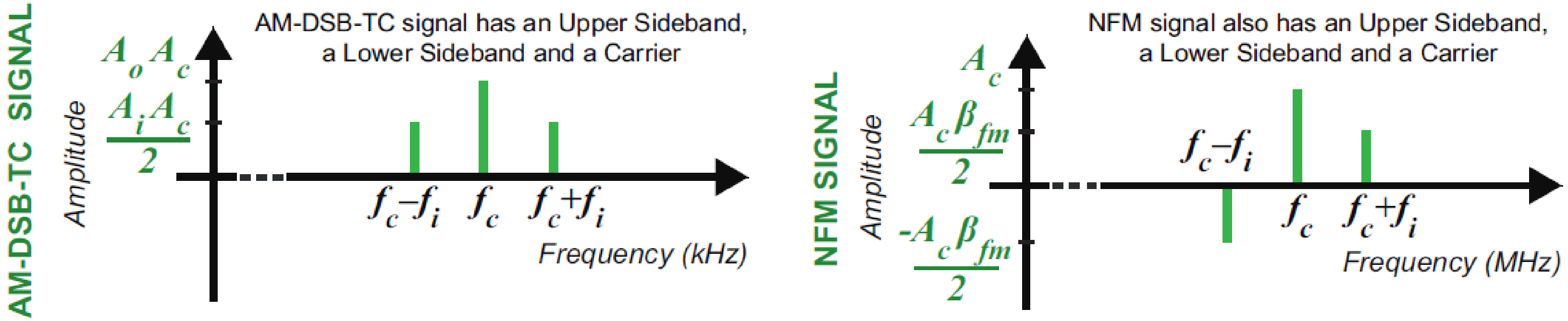
$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t) \right) = A_c \cos(\omega_c t) \cos\left(\beta_{fm} \sin(\omega_i t) \right) - A_c \sin(\omega_c t) \sin\left(\beta_{fm} \sin(\omega_i t) \right)$$

$$\cos\left(\beta_{fm} \sin(\omega_i t) \right) \approx 1 \quad and \quad \sin\left(\beta_{fm} \sin(\omega_i t) \right) \approx \beta_{fm} \sin(\omega_i t)$$

- Replacing back it will look similar to AM-DSB-TC

$$s_{fm-nfm}(t) = A_c \cos(\omega_c t) - A_c \sin(\omega_c t)\, \beta_{fm} \sin(\omega_i t)$$

$$= A_c \left[\cos(\omega_c t) + \frac{\beta_{fm}}{2} \cos(\omega_c + \omega_i)t - \frac{\beta_{fm}}{2} \cos(\omega_c - \omega_i)t \right]$$



WIDEBAND FM

- WFM is the standard used by commercial radio stations and it has a frequency deviation of 75kHz and a limited bandwidth of 200Khz.
- On WFM we have to solve:

$$\cos\left(\beta_{fm}\sin(\omega_i t)\right) \quad and \quad \sin\left(\beta_{fm}\sin(\omega_i t)\right)$$

- Using the Bessel we get:

$$\cos\left(\beta_{fm}\sin(\omega_i t)\right) = J_0(\beta_{fm}) + 2\sum_{n=1}^{\infty} J_{2n}(\beta_{fm})\cos(2n\omega_i t)$$

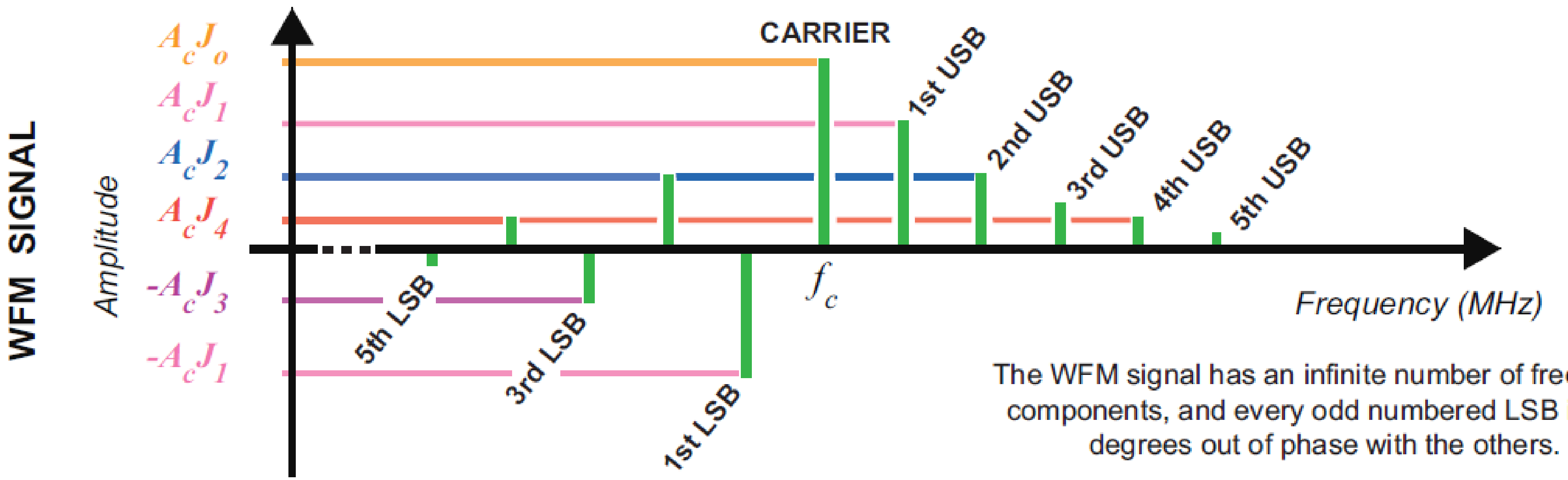
$$= J_0(\beta_{fm}) + 2J_2(\beta_{fm})\cos(2\omega_i t) + 2J_4(\beta_{fm})\cos(4\omega_i t) + \ldots$$

$$\equiv J_0 + 2J_2\cos(2\omega_i t) + 2J_4\cos(4\omega_i t) + \ldots$$

WIDEBAND FM

- If we consider just a tone to be transmitted:

$$
\begin{aligned}
s_{fm-wfm}(t) = {} & A_c J_0 \cos(\omega_c t) \\
& - A_c J_1 \left[\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t \right] \\
& + A_c J_2 \left[\cos(\omega_c - 2\omega_i)t + \cos(\omega_c + 2\omega_i)t \right] \\
& - A_c J_3 \left[\cos(\omega_c - 3\omega_i)t - \cos(\omega_c + 3\omega_i)t \right] \\
& + A_c J_4 \left[\cos(\omega_c - 4\omega_i)t + \cos(\omega_c + 4\omega_i)t \right] + \ldots
\end{aligned}
$$



The WFM signal has an infinite number of frequency components, and every odd numbered LSB is 180 degrees out of phase with the others.

WIDEBAND FM

- The bandwidth of a WFM signal can be estimated by finding the frequencies of the highest and lowest sideband components that contain a significant amount of power. $B = 2nf_i \text{ Hz}$
- Usually, we don't know **n** so it is estimated: $n = \beta_{fm} + 1$
- The BW is estimated using the Carlson Rule

$$B = 2(\beta_{fm} + 1) f_i$$

$$= 2\left(\frac{\Delta f}{f_i} + 1\right) f_i$$

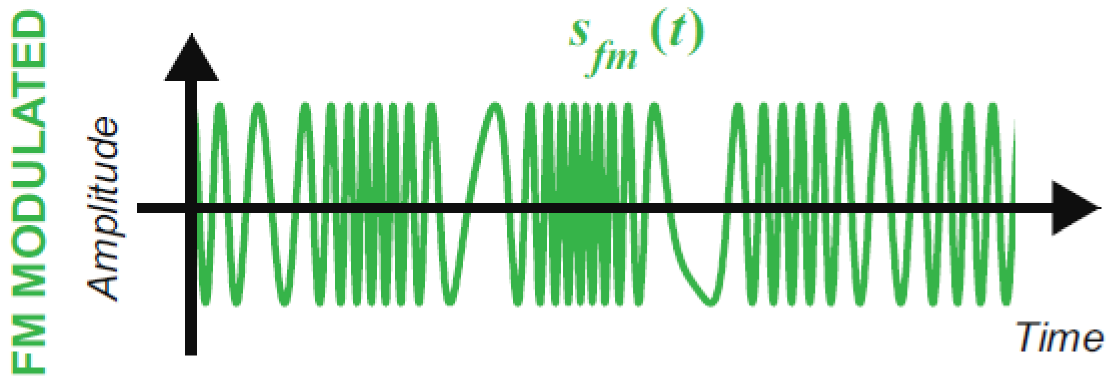
$$= 2(\Delta f + f_i) \text{ Hz}$$

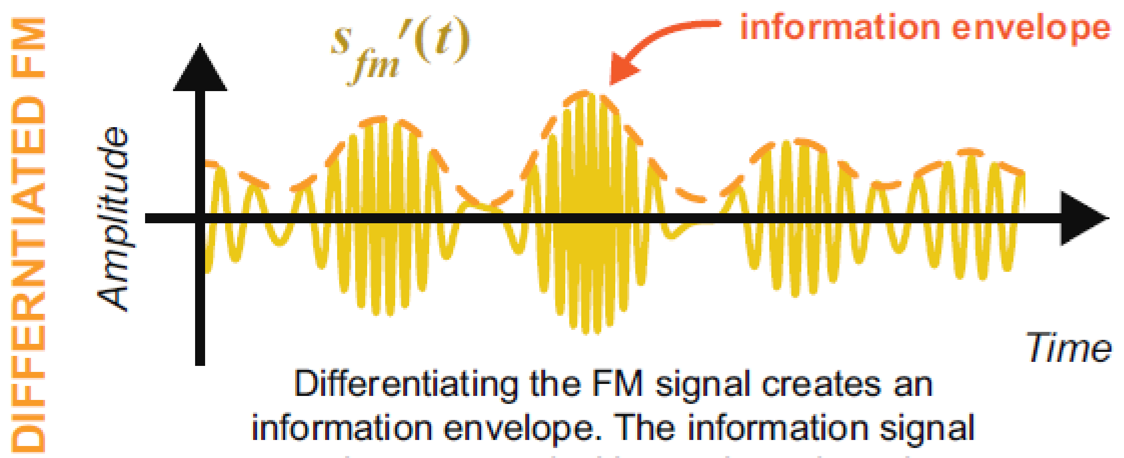
DIFFERENTIATOR DEMODULATOR

$$s_{fm}(t) = A_c \cos\left(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\,dt \right)$$

$$s_{fm}'(t) = \frac{d}{dt} s_{fm}(t) = -A_c \left[\omega_c + 2\pi K_{fm} s_i(t) \right] \sin\left(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\,dt \right).$$

amplitude component — *high frequency component*





Differentiating the FM signal creates an information envelope. The information signal can be recovered with envelope detection

The fluctuations in this envelope are directly proportional to the instantaneous frequency

RX COMPLEX BASEBAND

- The SDR has a quadrature downconverter. The baseband output signal look like:

$$s_{bband}(t) = s_{fmRF}(t)e^{-j\omega_{lo}t}$$

$$= s_{fmRF}(t) \times \left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

$$= A_c\cos\left(\omega_c t + \theta_{fm}(t)\right) \times \left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

$$s_{bband}(t) = \frac{A_c}{2}\left[\cos\left(\omega_c t + \theta_{fm}(t) - \omega_{lo}t\right) + \cos\left(\omega_c t + \theta_{fm}(t) + \omega_{lo}t\right)\right]$$

$$-j\frac{A_c}{2}\left[\sin\left(\omega_c t + \theta_{fm}(t) + \omega_{lo}t\right) - \sin\left(\omega_c t + \theta_{fm}(t) - \omega_{lo}t\right)\right]$$

high freq components

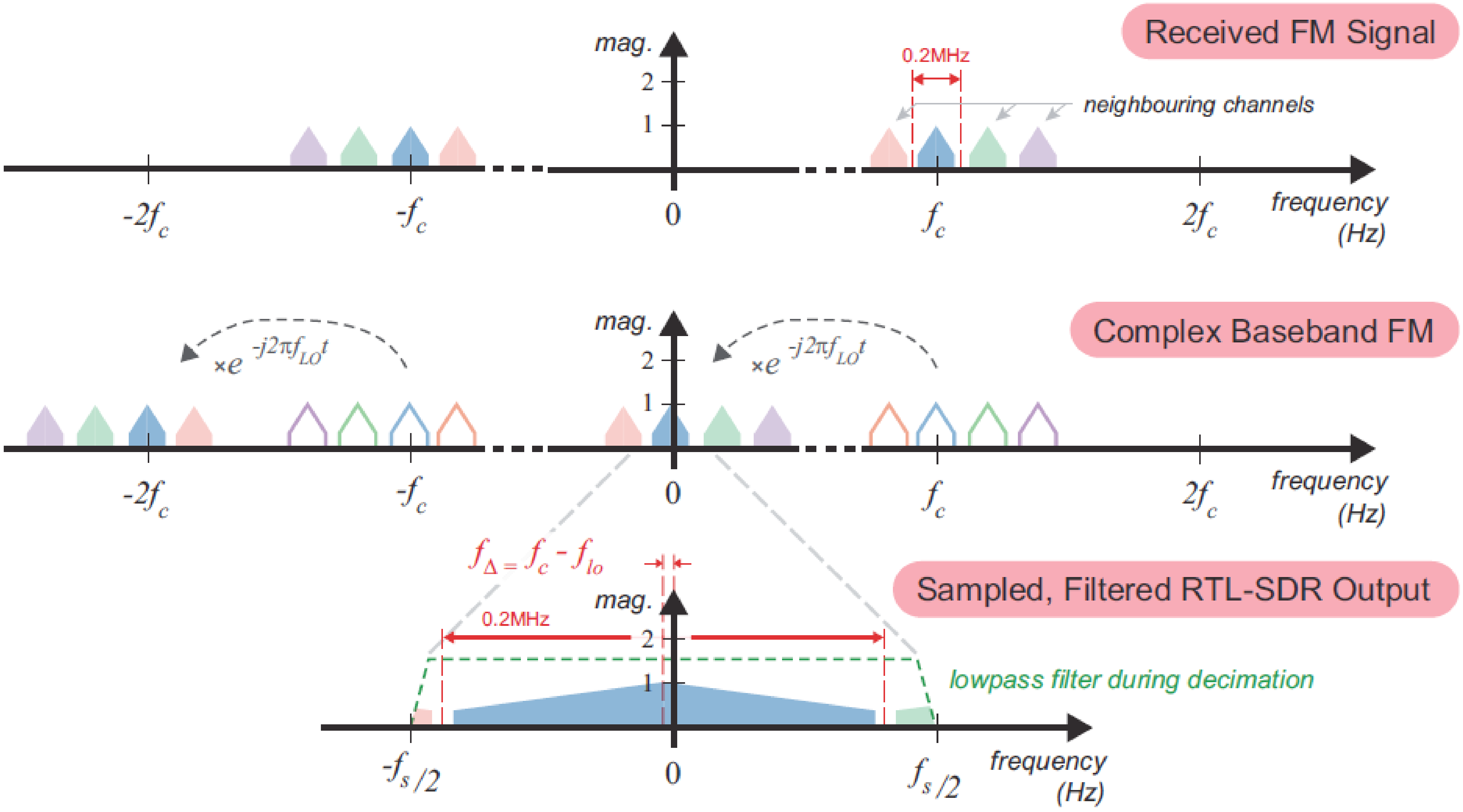
baseband components

- High frequency component removed by internal low pass filter.

- Close to zero $\omega_\Delta = \omega_c - \omega_{lo}$

$$s_{bband}(t) = s_{fmRF}(t)e^{-j\omega_{lo}} = \frac{A_c}{2}\left[\cos\left(\omega_\Delta t + \theta_{fm}(t)\right) + j\sin\left(\omega_\Delta t + \theta_{fm}(t)\right)\right] = \frac{A_c}{2}e^{j\left(\omega_\Delta t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt\right)}$$

COMPLEX BASEBAND RX



DEMO 1: COMPLEX DIFFERENTIATOR

- Differentiate both branch I/Q

$$\frac{A_c}{2}\left[\cos\left(\omega_\Delta t+\theta_{fm}(t)\right)+j\sin\left(\omega_\Delta t+\theta_{fm}(t)\right)\right]$$

$$s_{ip}'(t) = \frac{d}{dt}s_{ip}(t) = -\frac{A_c}{2}\left[\omega_\Delta+\theta_{fm}'(t)\right]\sin\left(\omega_\Delta t+\theta_{fm}(t)\right)$$

$$s_{qp}'(t) = \frac{d}{dt}s_{qp}(t) = \frac{A_c}{2}\left[\omega_\Delta+\theta_{fm}'(t)\right]\cos\left(\omega_\Delta t+\theta_{fm}(t)\right)$$

- Mixed them

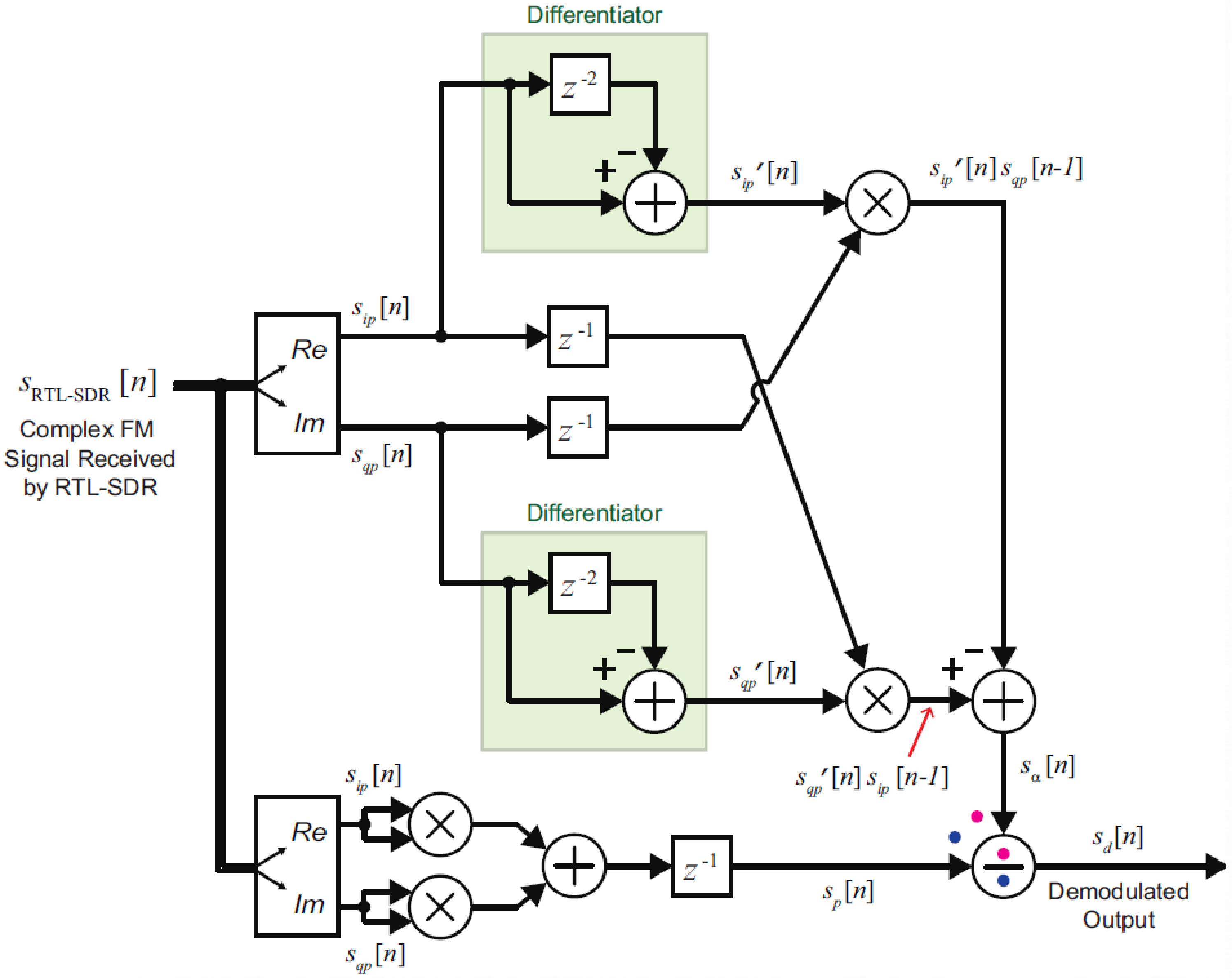
$$s_{ip}'(t)\times s_{qp}(t) = -\frac{A_c^2}{4}\left[\omega_\Delta+\theta_{fm}'(t)\right]\sin^2\left(\omega_\Delta t+\theta_{fm}(t)\right)$$

$$s_{qp}'(t)\times s_{ip}(t) = \frac{A_c^2}{4}\left[\omega_\Delta+\theta_{fm}'(t)\right]\cos^2\left(\omega_\Delta t+\theta_{fm}(t)\right).$$

- Subtract the terms

$$s_\alpha(t) = \left(s_{qp}'(t)\times s_{ip}(t)\right)-\left(s_{ip}'(t)\times s_{qp}(t)\right) = \frac{A_c^2}{4}\left[\omega_\Delta+\theta_{fm}'(t)\right]$$

DEMO 1: COMPLEX DIFFERENTIATOR



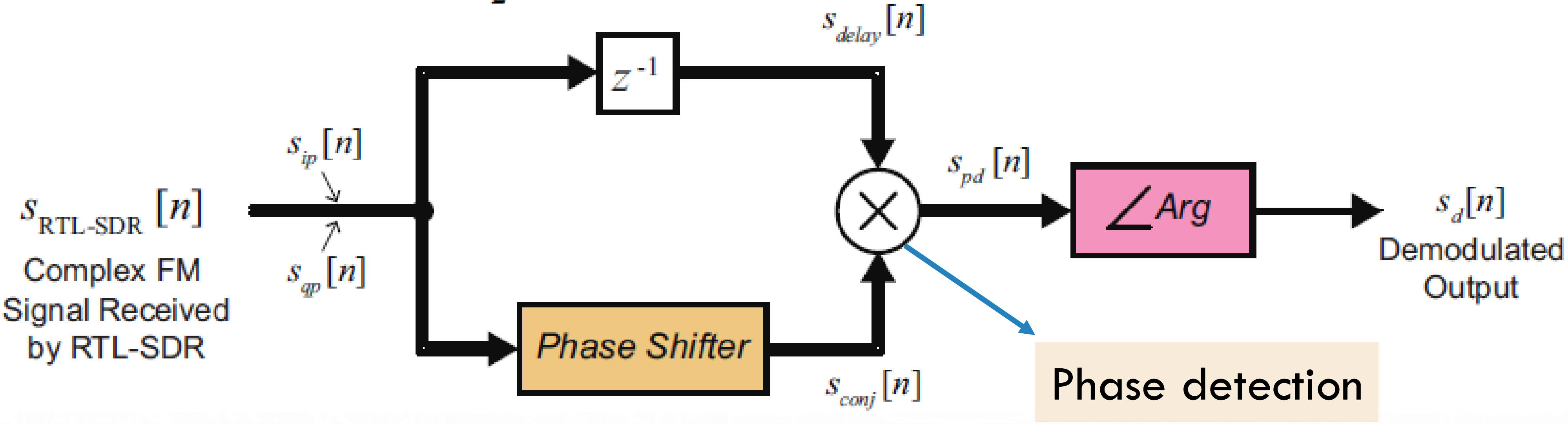
$$\theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt$$

$$s_n(t) = \frac{s_\alpha(t)}{s_p(t)} = \frac{\frac{A_c^2}{4}\left[\omega_\Delta + \theta_{fm}'(t)\right]}{\frac{A_c^2}{4}}$$

$$= \left[\omega_\Delta + \theta_{fm}'(t)\right]$$

DEMO 2: COMPLEX DELAY LINE

$$s_{delay}(t) = \frac{A_c}{2} e^{j\left(\omega_\Delta[t-\tau] + \theta_{fm}(t-\tau)\right)}$$



$$s_{conj}(t) = \frac{A_c}{2} e^{-j\left(\omega_\Delta t + \theta_{fm}(t)\right)}$$

$$s_{pd}(t) = s_{conj}(t) \times s_{delay}(t)$$

$$= \frac{A_c^2}{4} e^{-j\left[\left(\omega_\Delta t + \theta_{fm}(t)\right) - \left(\omega_\Delta[t-\tau] + \theta_{fm}(t-\tau)\right)\right]}$$

DEMO 2: COMPLEX DELAY LINE

- Taking the angle of the signal

$$s_d(t) = \angle s_{pd}(t) = -\left[\left(\omega_\Delta t + \theta_{fm}(t)\right) - \left(\omega_\Delta[t-\tau] + \theta_{fm}(t-\tau)\right)\right]$$

$$= -\left[\left(\omega_\Delta t - \omega_\Delta[t-\tau]\right) + \left(\theta_{fm}(t) - \theta_{fm}(t-\tau)\right)\right]$$

- If the delay is small

$$s_d(t) \approx -\left[\frac{d}{dt}(\omega_\Delta t) + \frac{d}{dt}\left(\theta_{fm}(t)\right)\right]$$

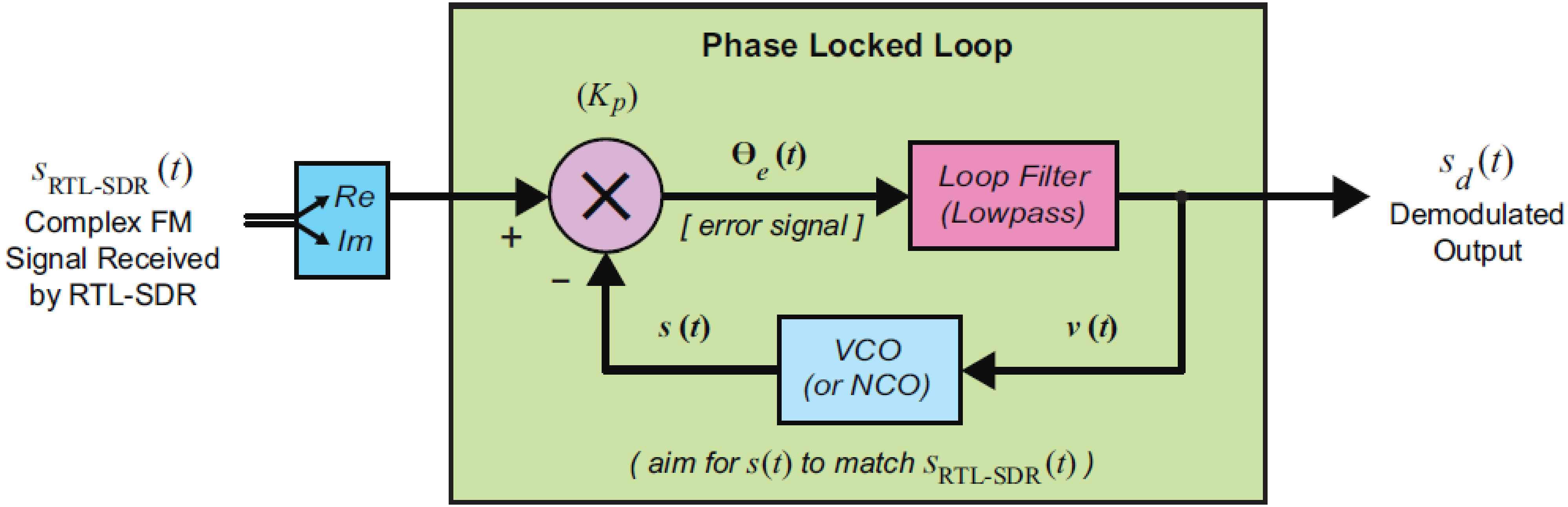
$$s_d(t) = -\left[\omega_\Delta + \theta_{fm}'(t)\right]$$

$$= -\left[\omega_\Delta + 2\pi K_{fm} s_i(t)\right]$$

- Similar result as the complex differentiator, but simpler implementation.
- If you want to implement on FPGA, you have to use CORDIC algorithms to make efficient.

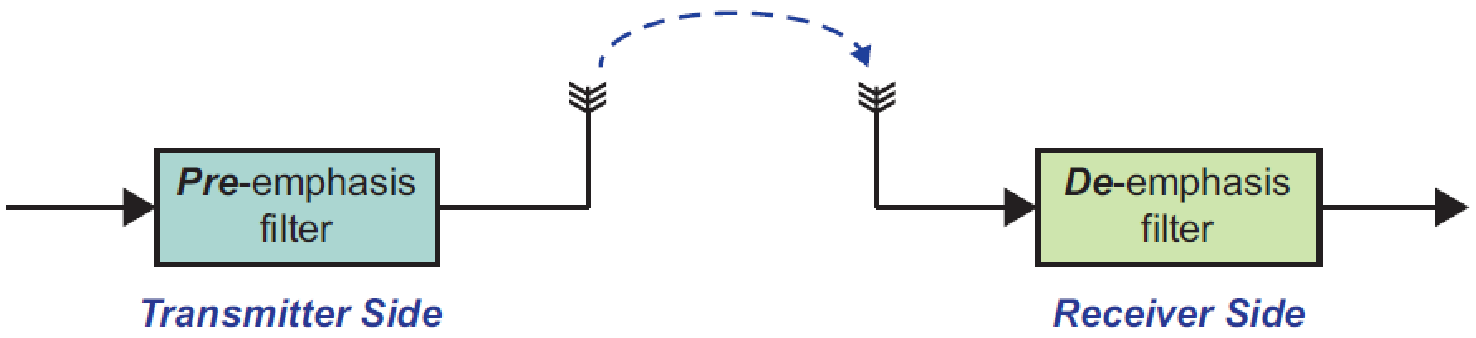
DEMO 3: PLL COHERENT RECEIVER

- PLL will try to follow the frequency changes, it will never lock.
- To function as an FM demodulator, the internal parameters of the PLL must be chosen appropriately.



COMMERCIAL RADIOS

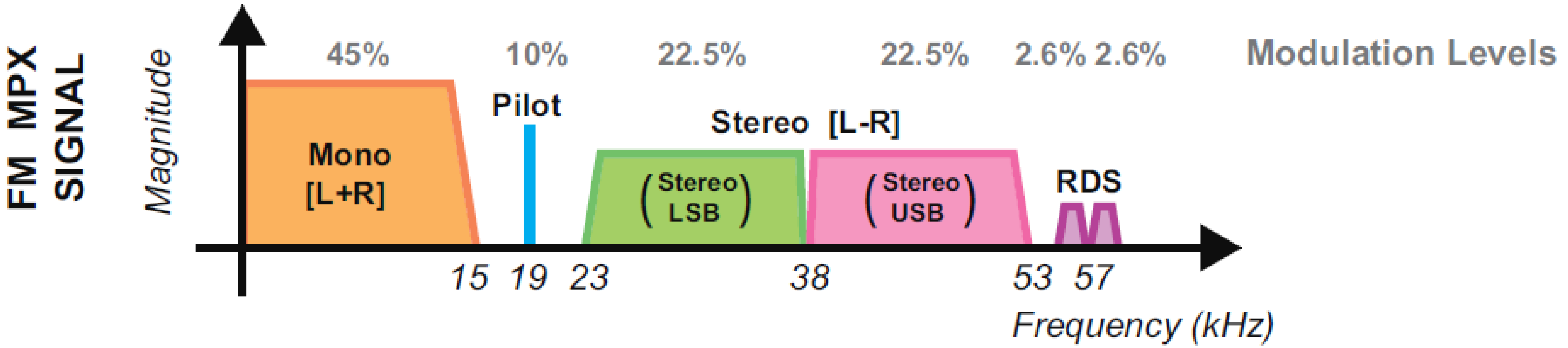
- Pre-emphasis & De- Emphasis, it is done to give gain to high frequency components in order to maintain the station bandwidth.
- For US is a filter with a time constant of 75us and for Europe 50us.



- It is implemented with an analog filter.

COMMERCIAL RADIOS

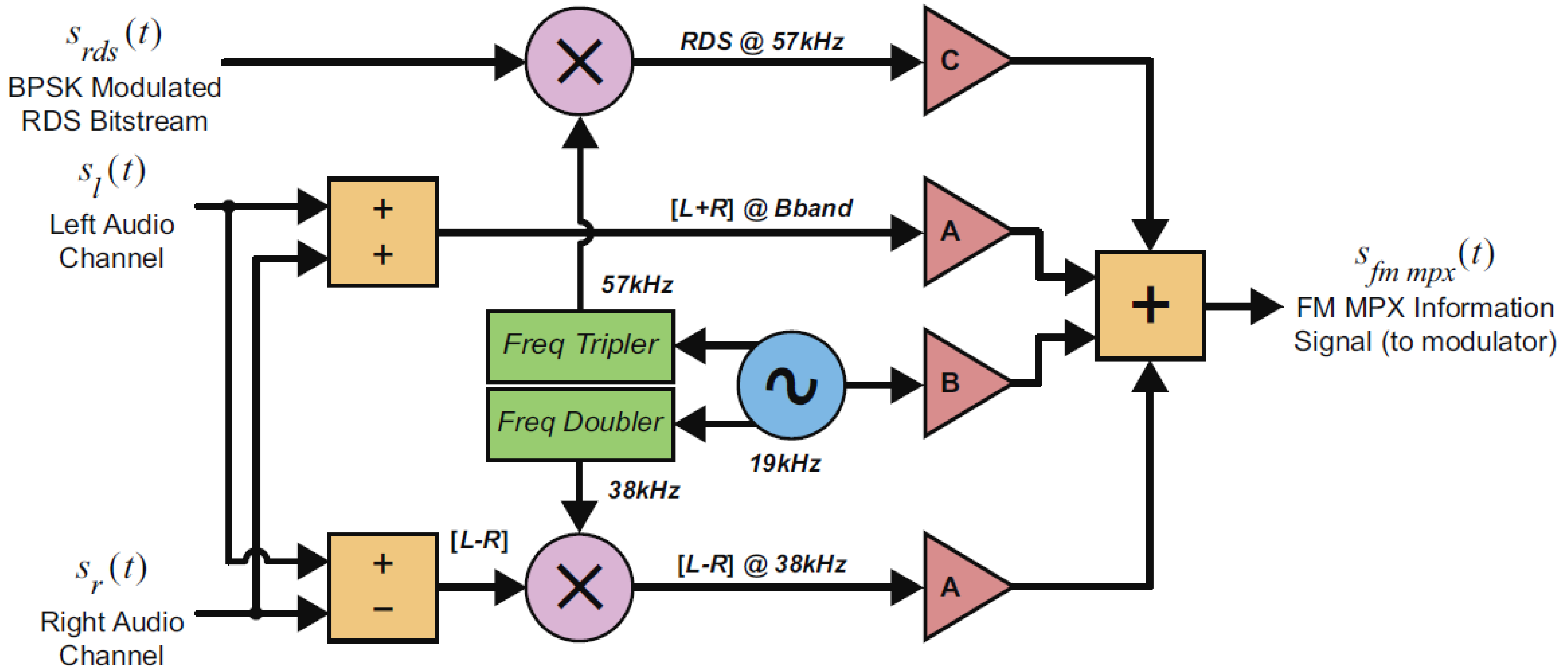
- MULTIPLEX: today stations still transmit MONO signals for all receivers, and STEREO + RDS signal for new receivers.



- MONO= L+R
- Stereo= is modulated as AM-DSB-SC a 38Khz
- The pilot inform is there is STEREO information, and is used to extract carrier information for coherent demo.
- RDS=Radio Data System, used to transmit the song track, station info.

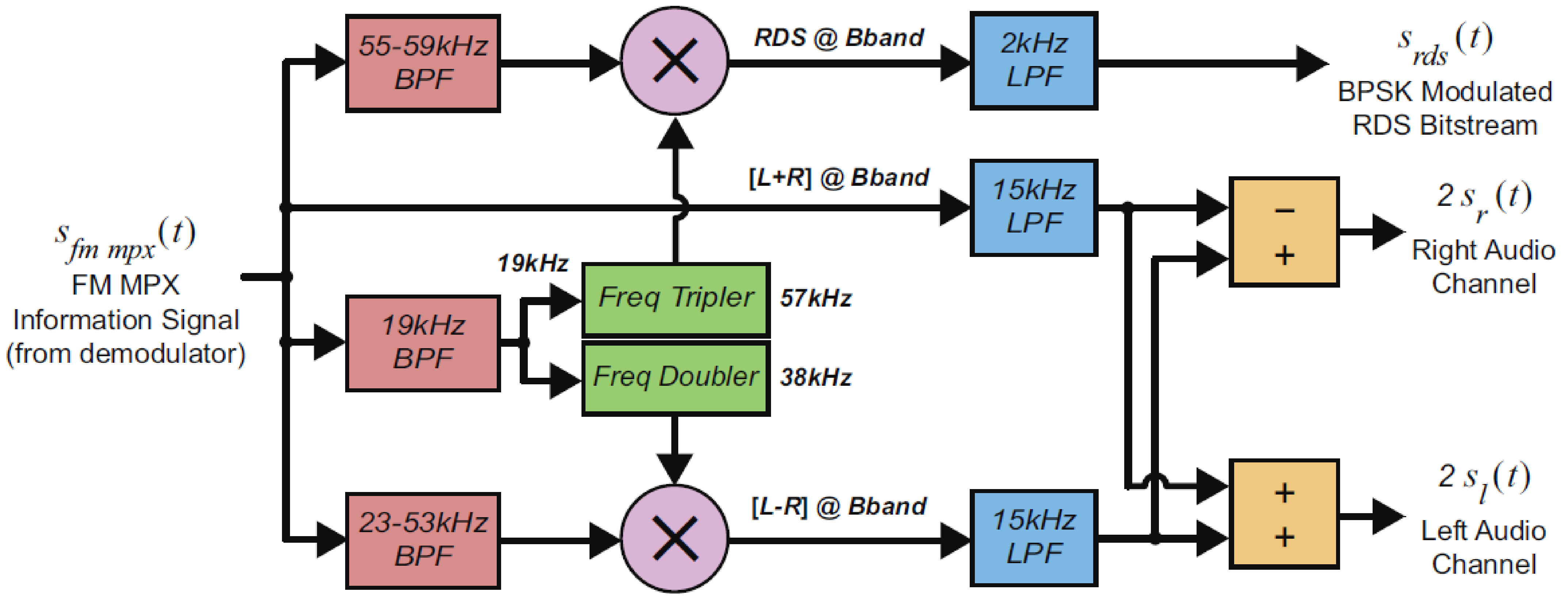
COMMERCIAL RADIOS

- MULTIPLEXER TX



COMMERCIAL RADIOS

- DE-MULTIPLEX on RX



FM BASEBAND REPRESENTATION

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \theta_{fm}(t)\right) \quad where \quad \theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt$$

$$s_{fm-baseband}(t) = A_c e^{-j\theta_{fm}(t)} = A_c \angle -\theta_{fm}(t)$$

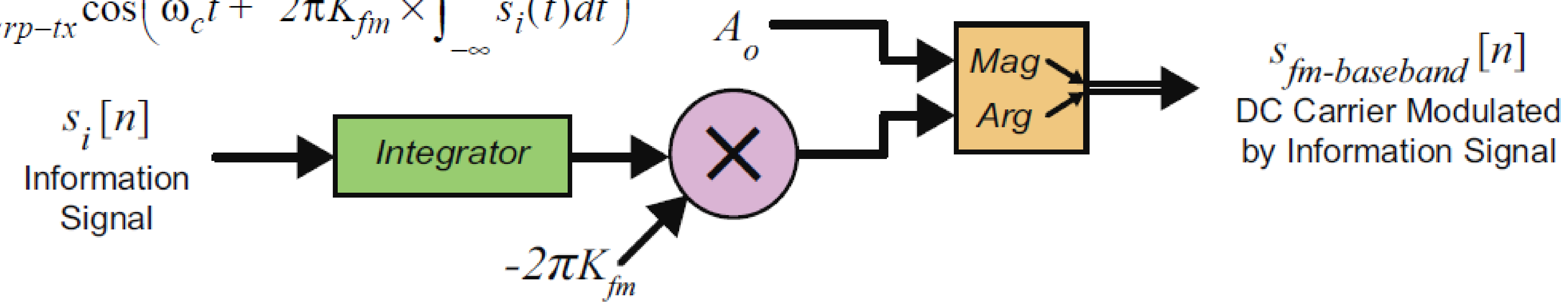
$$s_{usrp-tx}(t) = K_{usrp-tx}\left[\Re e\left(s_{fm-baseband}(t)\right)\cos(\omega_c t) + \Im m\left(s_{fm-baseband}(t)\right)\sin(\omega_c t)\right]$$

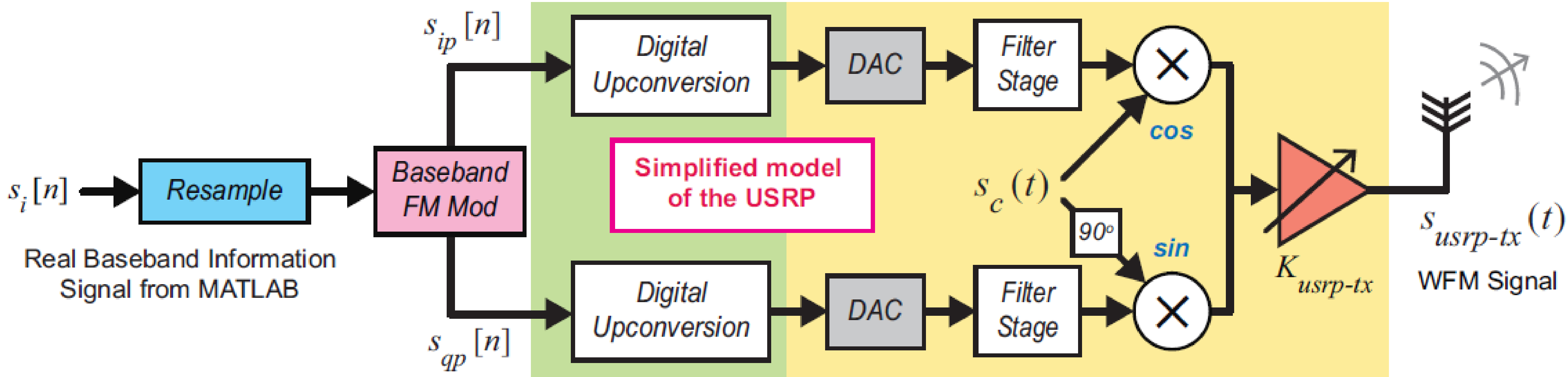
$$= K_{usrp-tx}\left[A_c \cos\left(-\theta_{fm}(t)\right)\cos(\omega_c t) + A_c \sin\left(-\theta_{fm}(t)\right)\sin(\omega_c t)\right].$$

$$= A_c K_{usrp-tx} \cos\left(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt\right)$$

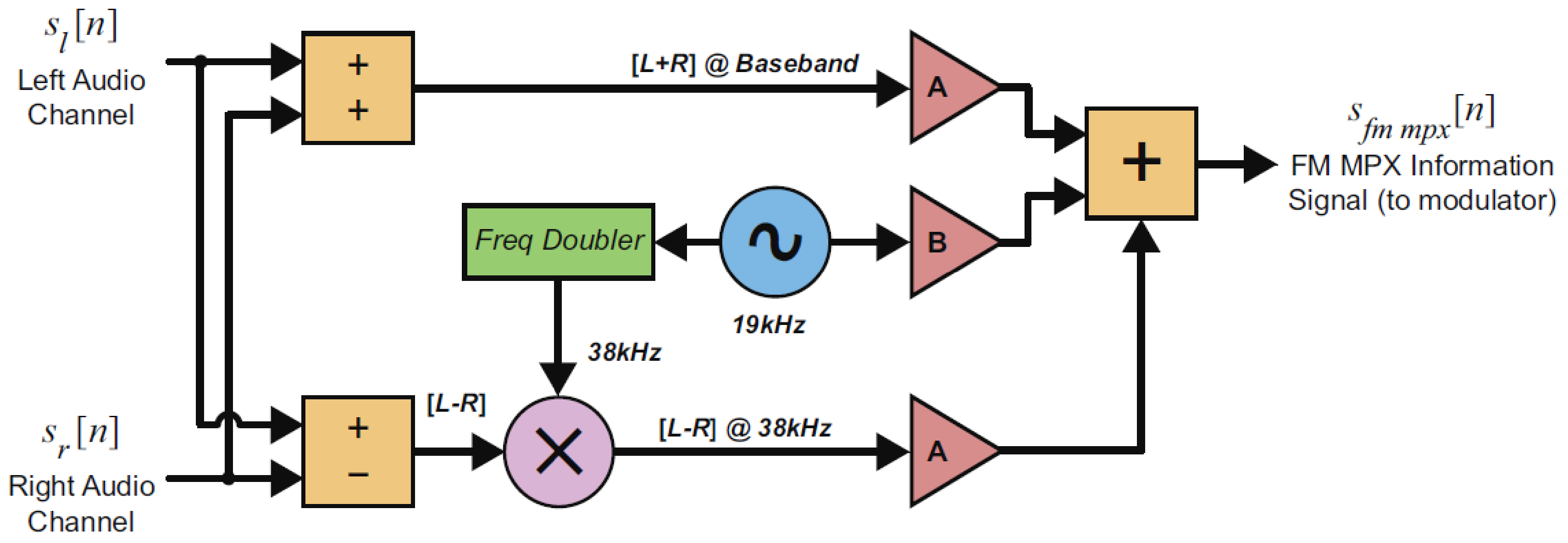
FM MONO MODULATOR

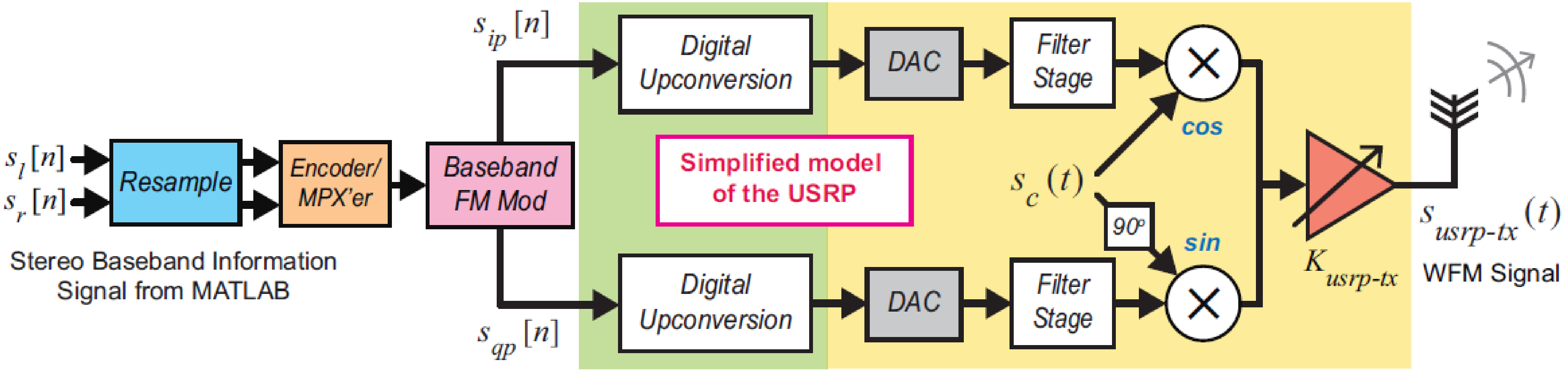
$$= A_c K_{usrp-tx} \cos\left(\omega_c t + \; 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt\right)$$



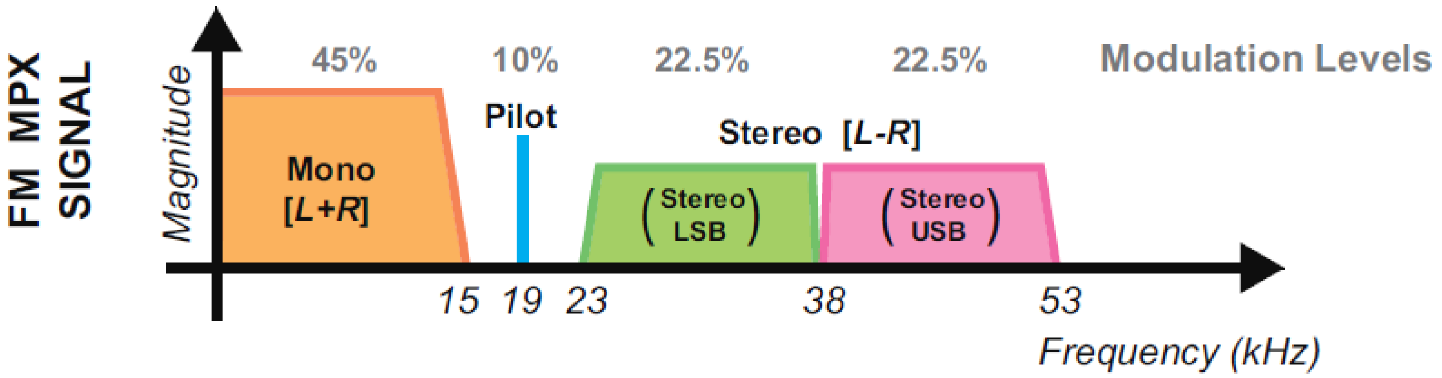


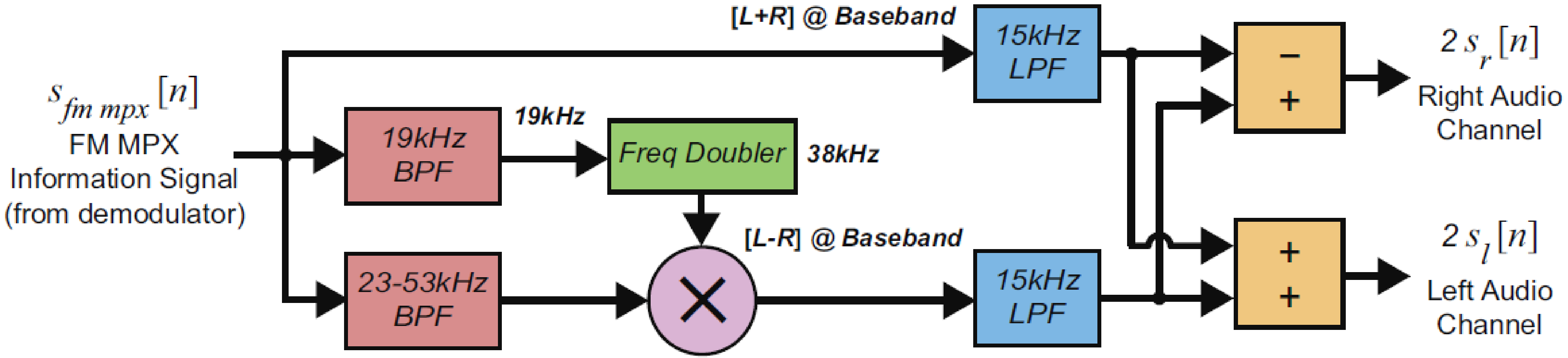
FM STEREO MODULATOR





FM STEREO DEMODULATOR





RX SIGNAL

REAL FM MPX SIGNAL RECEIVED WITH THE RTL-SDR