Risk-Aware Control of IoT-Enabled Systems with Imperfect Distributional Information about Uncertainties

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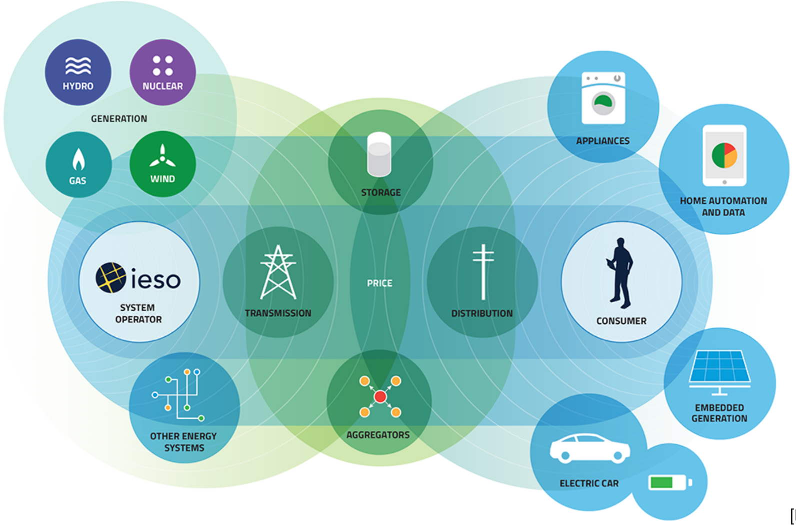
Trends in IoT-enabled systems

- Physical systems with sensing, communication and computing capabilities (e.g., smart grids)



[IESO]

Trends in IoT-enabled systems

- Physical systems with sensing, communication and computing capabilities
- Data-driven decision-making (e.g., smart home)

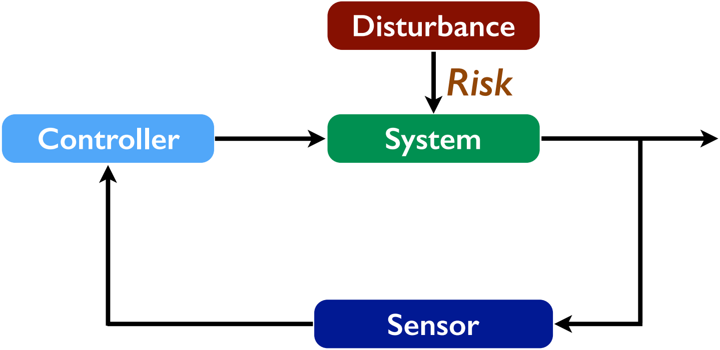
[NEST]

Trends in IoT-enabled systems

- Physical systems with sensing, communication and computing capabilities
- Data-driven decision-making
- Autonomy or semi-autonomy (e.g., drone air traffic control)

[NASA]

From traditional to IoT-enabled systems: Risk generated from data, statistics, human



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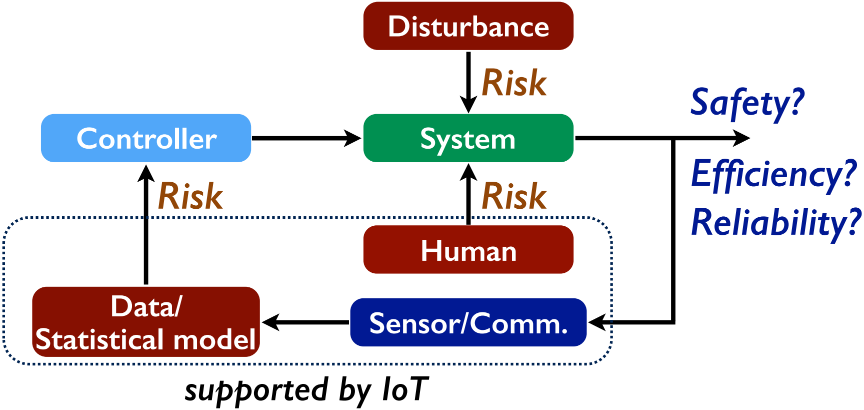

- Data: inaccuracy, insufficient samples
- Statistical model: wrong prior knowledge, local optimality

Risk-aware distributionally robust control

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robust against errors in the probability distribution of uncertainties?

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$$\text{s.t.} \quad \text{Risk}^{\mu}[\text{Loss}(x,u,w)] \le R \quad \textbf{(Worst-case risk)}$$

$$x_{t+1} = f(x_t, u_t, w_t), \quad w_t \sim \mu_t \quad \text{(System dynamics)}$$

$$\mu_t \in \mathbb{D}_t \quad \textbf{(Admissible prob. distributions)}$$

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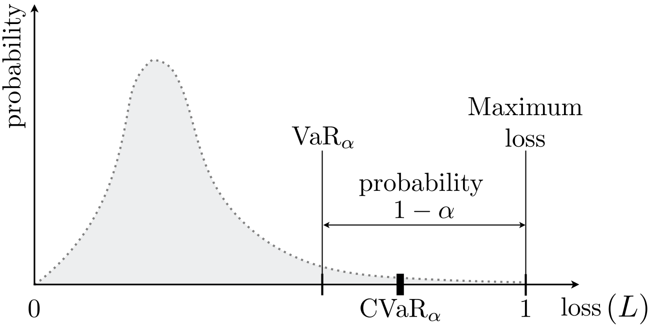
- Use of risk measures: chance constraints, variance, CVaR
- Consideration of the worst-case probability distribution of uncertainties: **allowing errors** *in estimated distributional information about uncertainties*

Examples of risk measures



Value-at-Risk

$$\mathrm{VaR}_\alpha(L) := \inf\{x \in \mathbb{R} \mid \underbrace{F_L}_{\text{c.d.f of } L}(x) \geq \alpha\}$$

Conditional Value-at-Risk

$$\mathrm{CVaR}_\alpha(L) := \mathbb{E}[L \mid L \geq \mathrm{VaR}_\alpha(L)]$$

"**Penalizing the** $(1-\alpha)$ **worst-case quantile**"

Examples of ambiguity sets of admissible distributions

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- Moment constraints ($\mathbf{m}_t$, Σ_t: mean and covariance estimate)

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Examples of ambiguity sets of admissible distributions

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- Statistical distance (ν: empirical (nominal) distribution)

$$\mathbb{D}_t := \big\{ \mu_t \in \mathcal{P}(\mathbb{R}^l) \mid W_p(\mu_t, \nu) \leq \theta \big\}$$

Dynamic programming solution

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$$\begin{aligned}\min_u \max_\mu \quad & \mathbb{E}^\mu[\mathsf{Cost}(x,u,w)] + \lambda \mathsf{Risk}^\mu[\mathsf{Loss}(x,u,w)] \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t, w_t), \quad w_t \sim \mu_t \\ & \mu_t \in \mathbb{D}_t\end{aligned}$$

- Risk as expectation minimization (for a class of risk measures)
  - CVaR: $\mathsf{CVaR}_\alpha[L] = \min_{y\in\mathbb{R}} \mathbb{E}\big[y + \frac{1}{1-\alpha}(L-y)^+\big]$
  - Variance: $\mathsf{Var}[L] = \min_{y\in\mathbb{R}} \mathbb{E}[(L-y)^2]$
  - Median absolute deviation: $\mathsf{MAD}[L] = \min_{y\in\mathbb{R}} \mathbb{E}[|L-y|]$

Dynamic programming solution

$$\begin{aligned} \min_{u} \max_{\mu} \quad & \mathbb{E}^{\mu}[\mathsf{Cost}(x,u,w)] + \lambda \min_{y \in \mathbb{R}^k} \mathbb{E}^{\mu}[g(\mathsf{Loss}(x,u,w), y)] \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t, w_t), \quad w_t \sim \mu_t \\ & \mu_t \in \mathbb{D}_t \end{aligned}$$

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- Bilevel optimization formulation: **[Miller, Yang, SIAM J. Control and Optimization, 2017]**
  - inner "minimax" problem (over u, μ) – dynamic programming
  - outer "min" problem (over y) – gradient descent

Dynamic programming solution

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- Risk as expectation minimization for a class of risk measures (e.g., CVaR, variance, MAD)
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- Duality-based dynamic programming: **[Yang, arXiv:1701.06260, 2017]**
  - Strong duality in infinite dimensional LP
  - Semi-infinite program formulation

Application to smart home energy management

- Objective: energy cost-minimizing air conditioning
- Risk constraint: Prob(temperature in comfort range) ≥ 0.95
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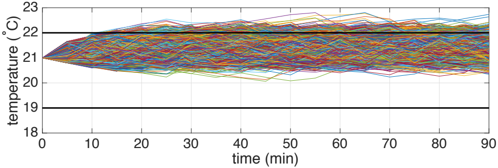
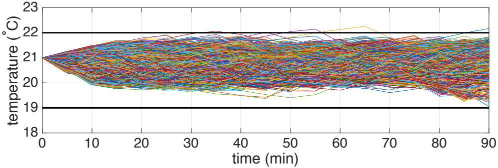
1. Standard probabilistic safety-aware control (Prob. of safety: 0.86)



2. Proposed distributionally robust control (Prob. of safety: 0.995)



Ongoing and future work

- Improving scalability
- Combining with statistical learning for real-time adaptation
- Applications:
  - refrigerator energy and inventory control under demand uncertainty
  - smart home and building
  - smart grids (balancing uncertain wind energy)
  - air traffic management for drones
  - semi-autonomous systems with uncertain human inputs and preferences

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New special topic course: EE 599 Data-Driven Optimization and Control (Fall 2017)