Modeling, Dynamics, and Control of Cyber-Physical Systems

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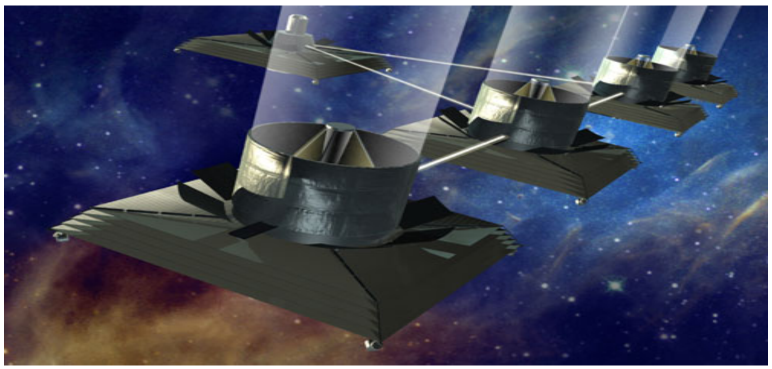
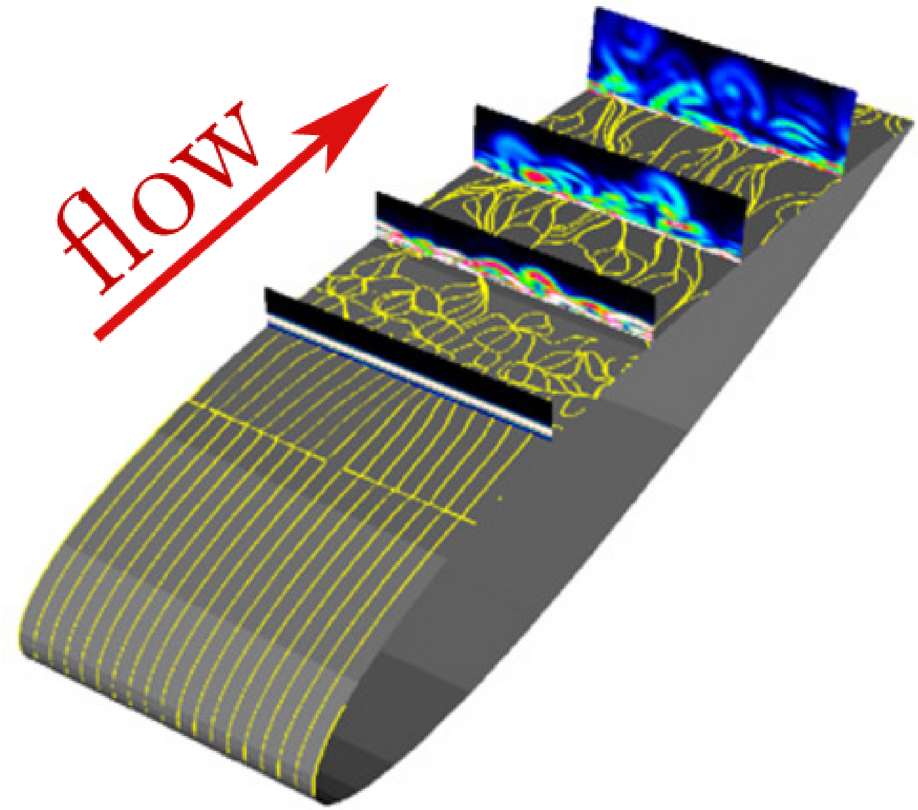
Mini-Workshop on Theoretical Foundations of CPS

Motivating applications

networks of dynamical systems



fluid flows



- **Modeling, dynamics, and control of distributed systems**
  - ⋆ theory and applications
  - ⋆ methods for uncertainty propagation, analysis, and design

Inter-area oscillations in power systems

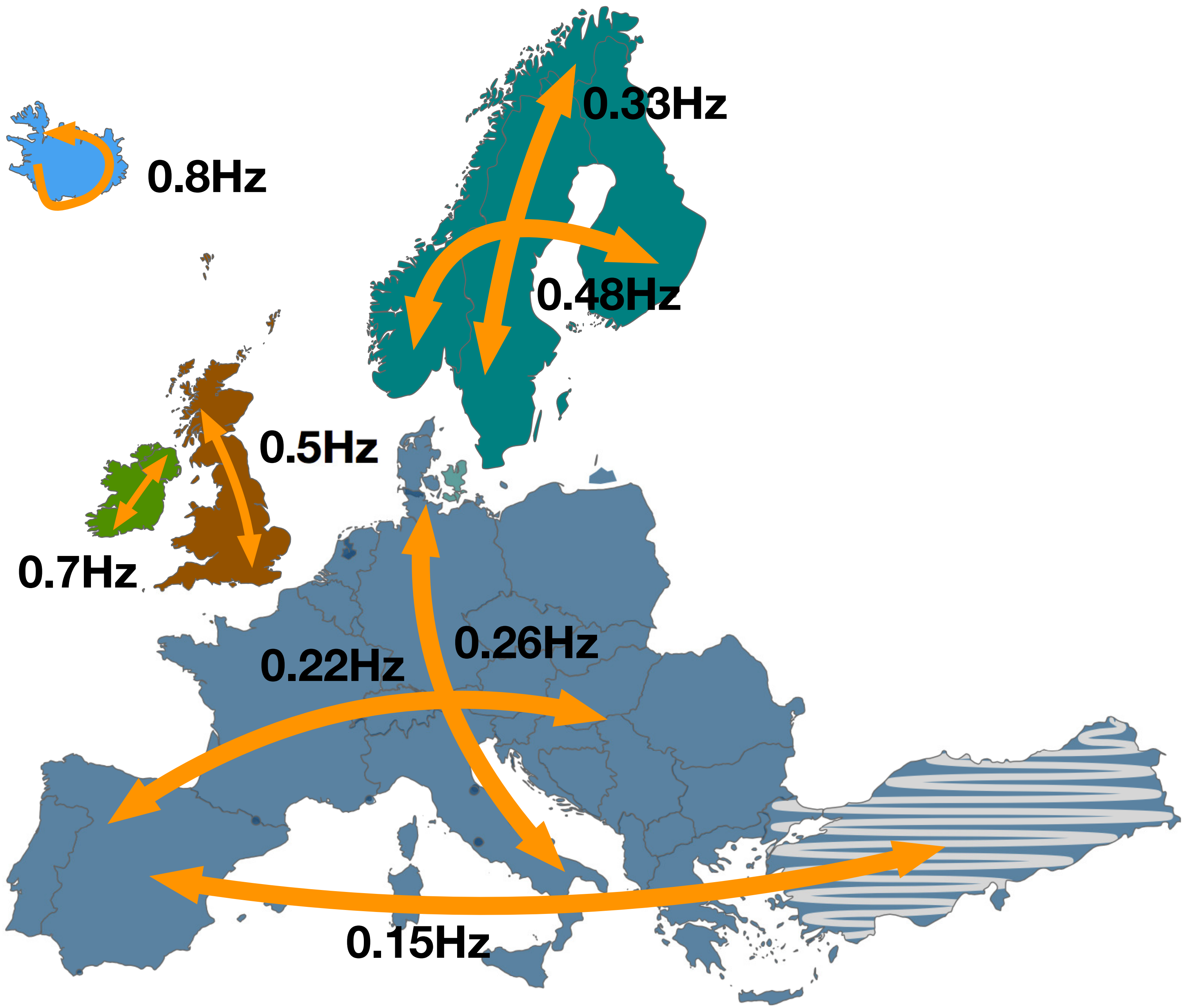
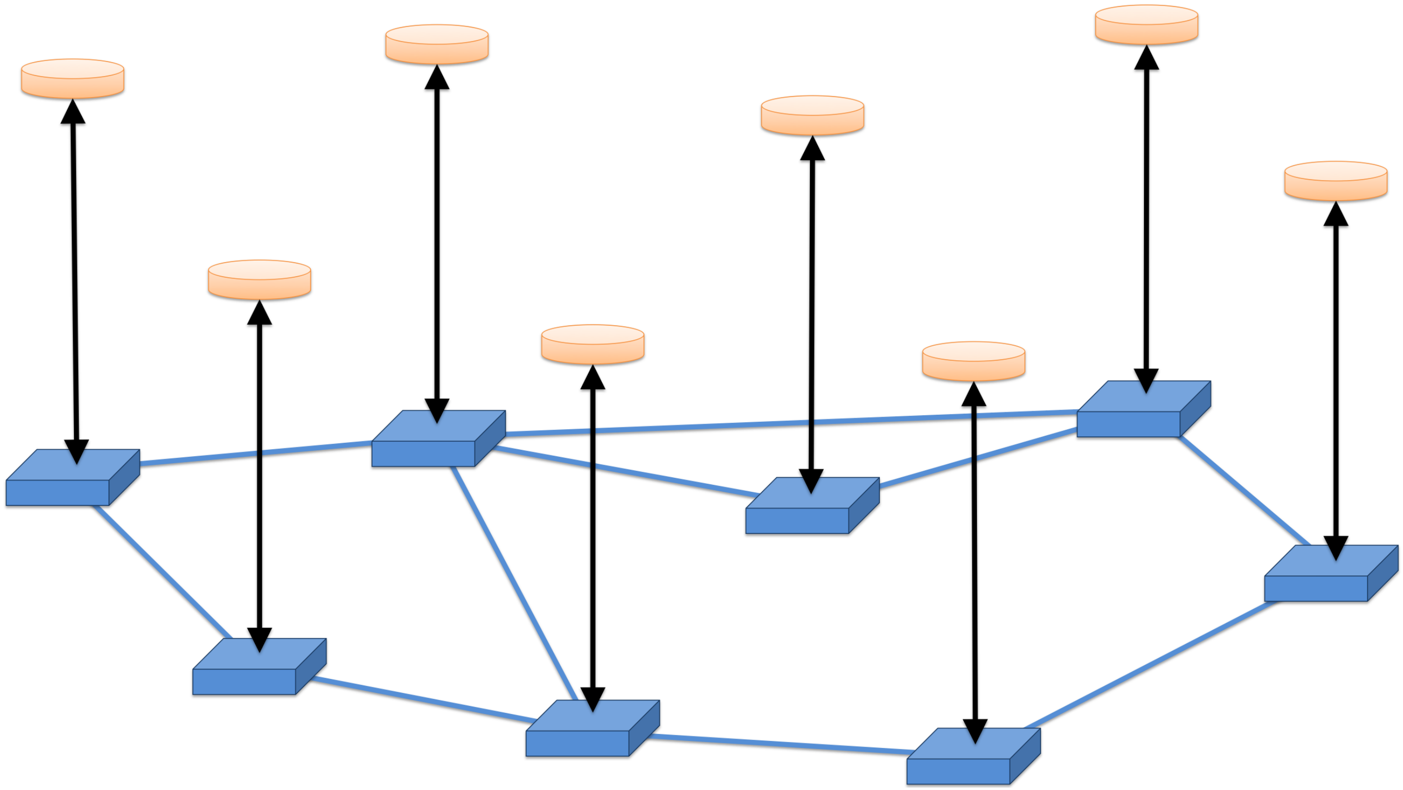


Image credit: Florian Dörfler

CONTROL

Conventional control of generators

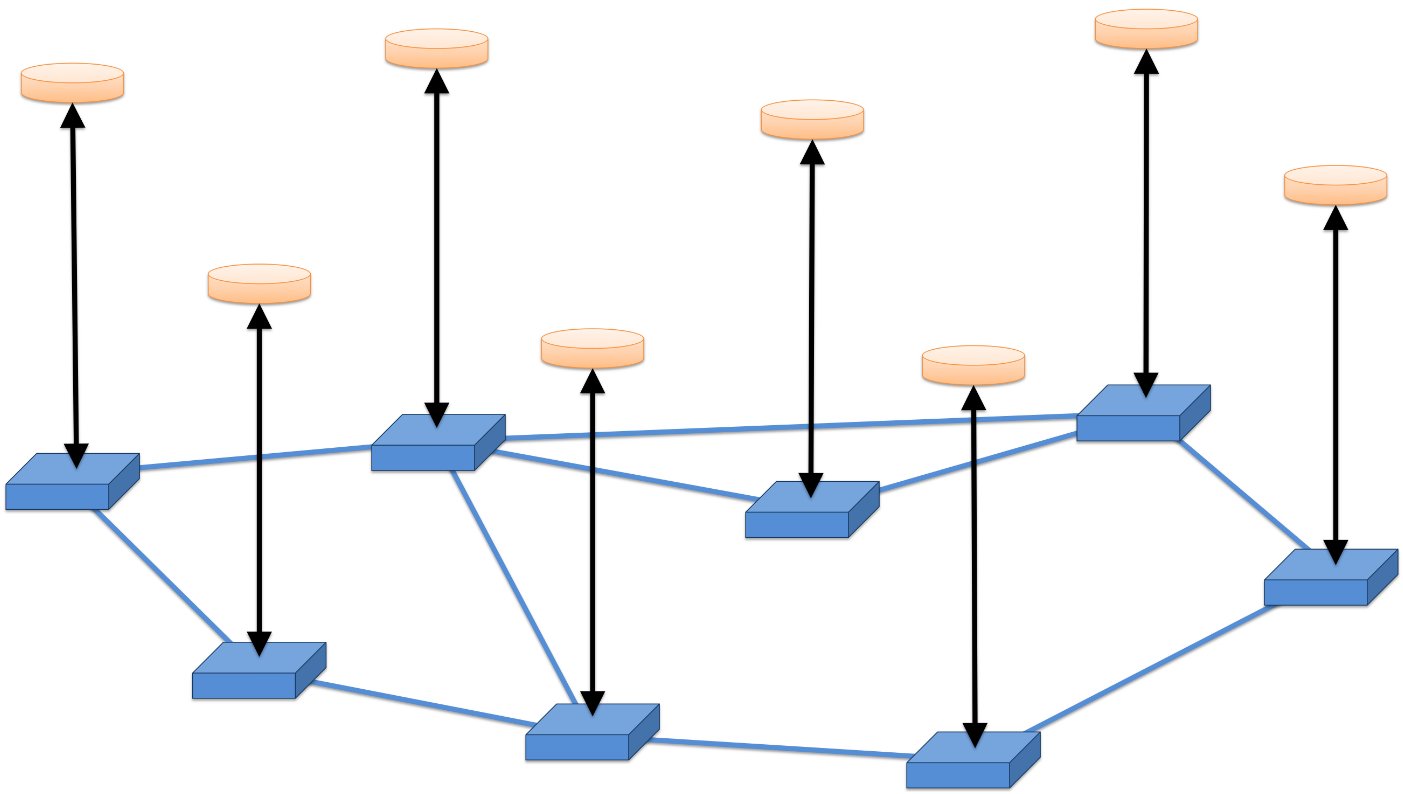
fully decentralized controller



network of generators

Conventional control of generators

fully decentralized controller

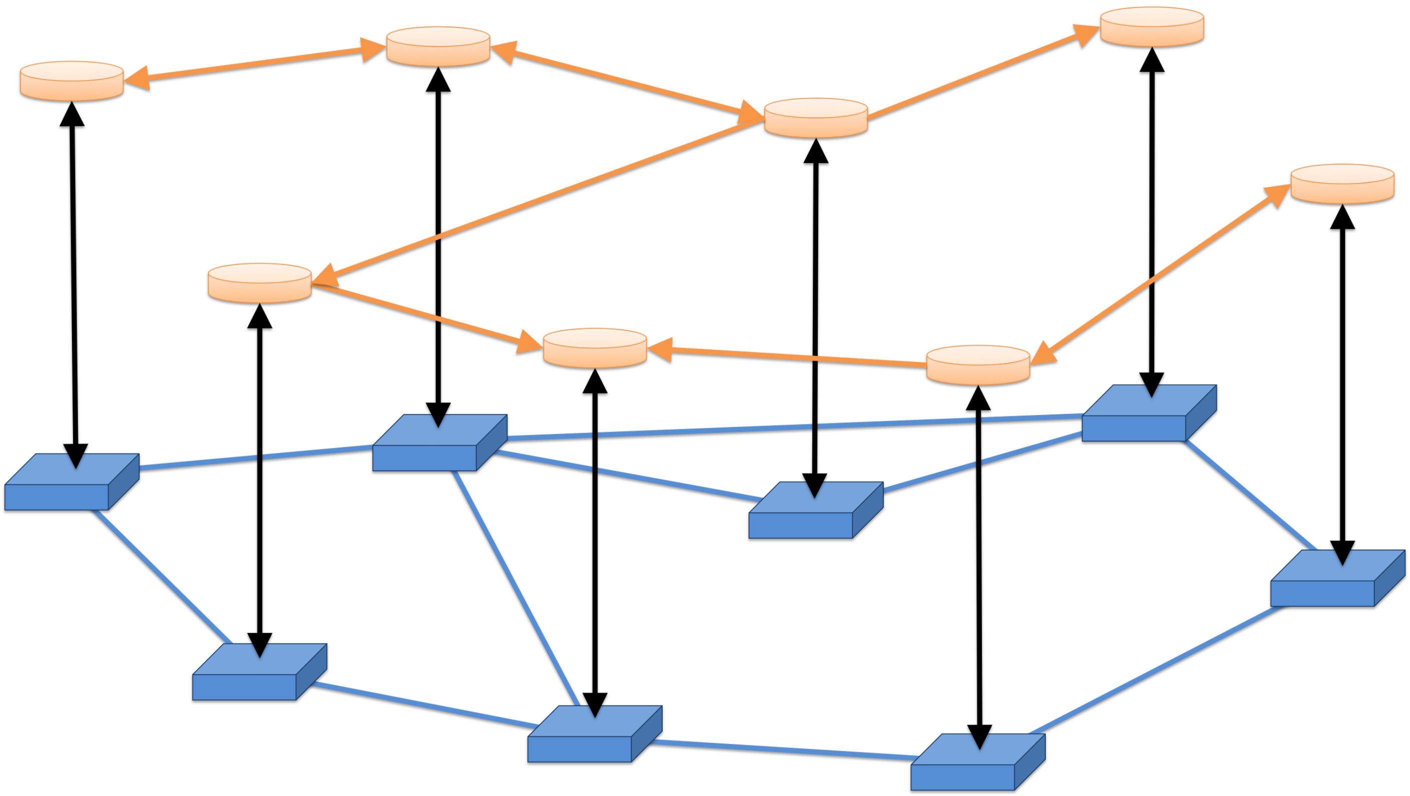


network of generators

- CONVENTIONAL CONTROL
  - ⋆ local oscillations ✔
  - ⋆ inter-area oscillations ✘

Possible alternative

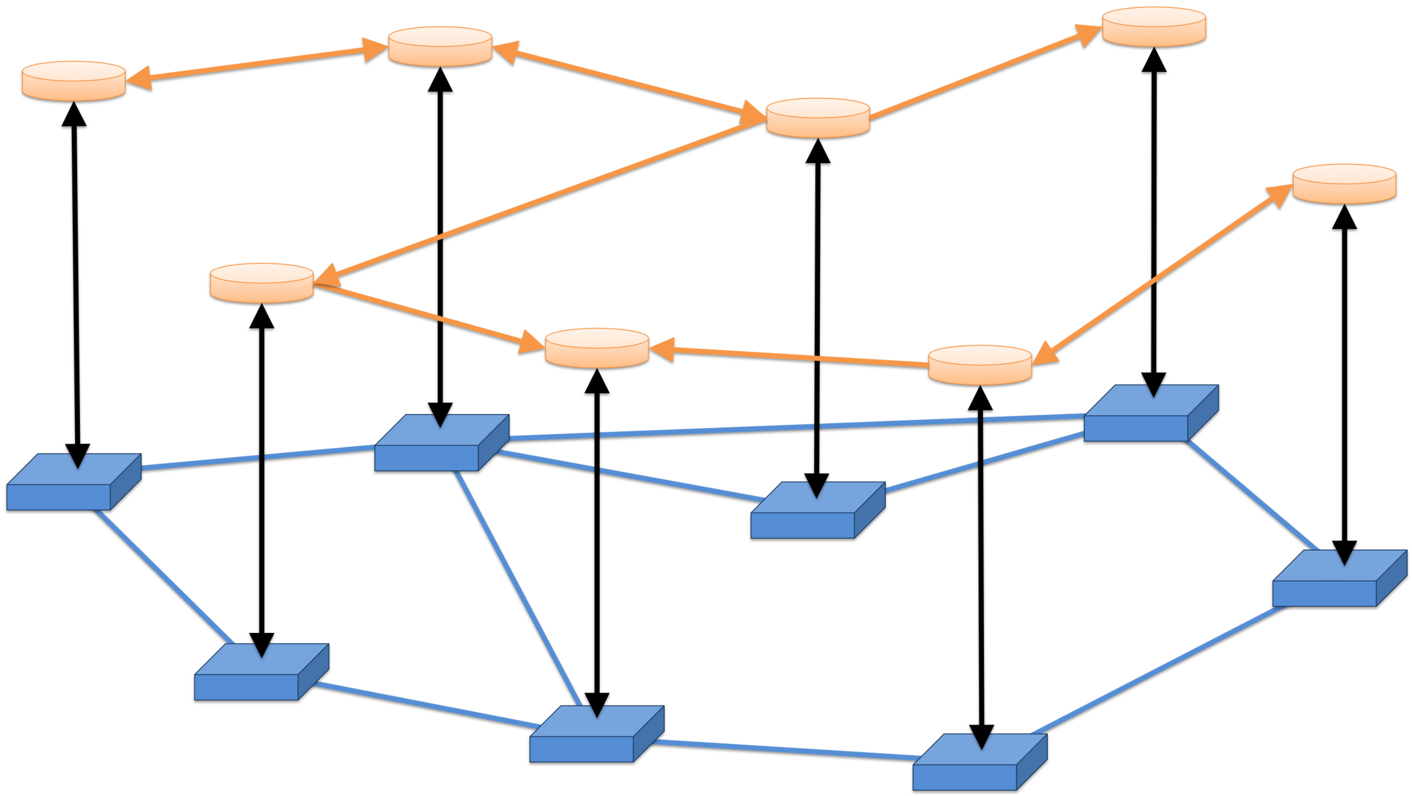
structured dynamic controller



distributed plant and its interaction links

Possible alternative

structured dynamic controller



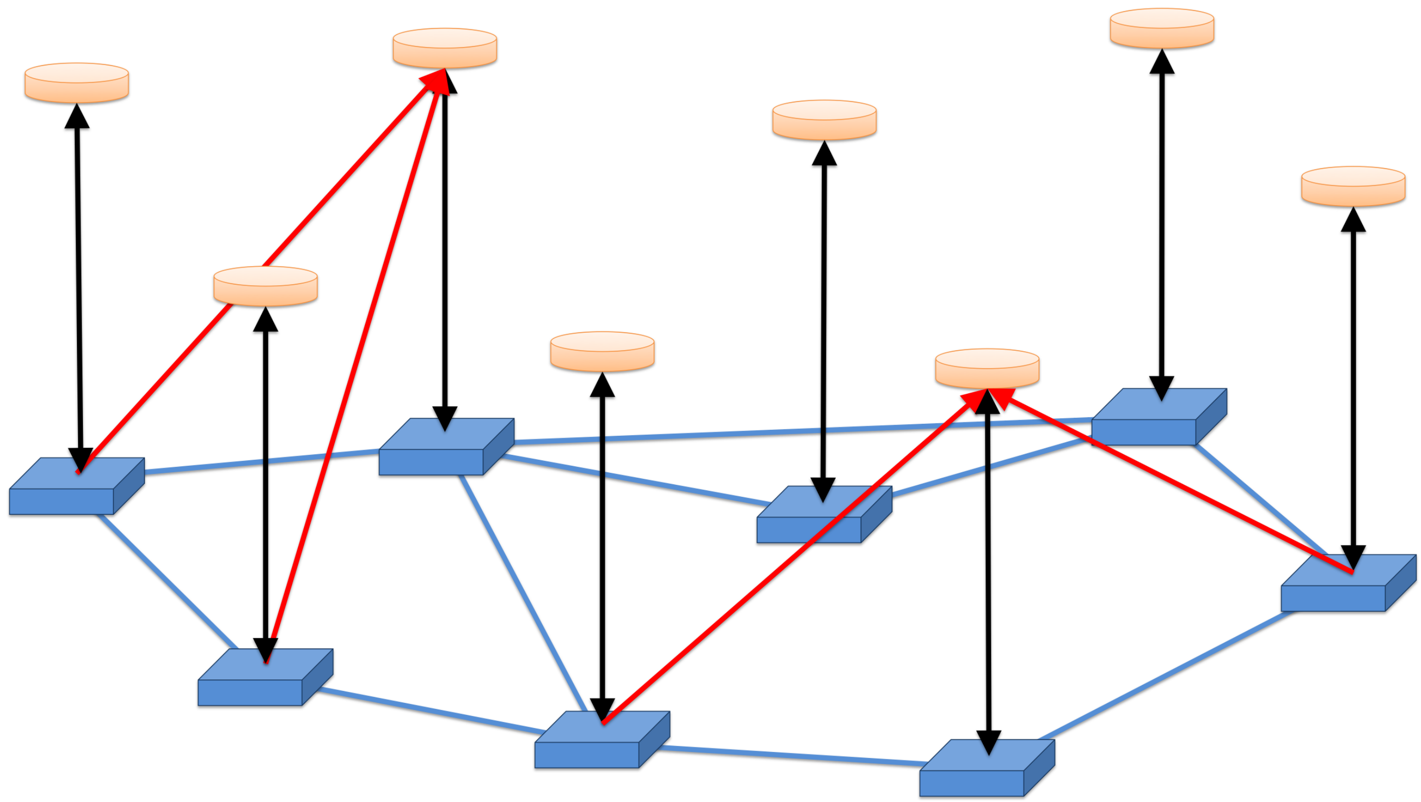
distributed plant and its interaction links

CHALLENGE

design of controller architectures

performance vs **complexity**

structured memoryless controller



distributed plant and its interaction links

OBJECTIVE

identification of a signal exchange network

performance vs **sparsity**

• OBJECTIVE

⋆ **promote sparsity of feedback gain F**

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix} = - \underbrace{\begin{bmatrix} * & * & & & \\ * & * & & & * \\ & * & * & * & \\ & & * & * & \\ & * & & * & * \end{bmatrix}}_{F} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}$$

Sparsity-promoting optimal control

$$\text{minimize} \quad J(F) \quad + \quad \gamma \sum_{i,\,j} w_{ij}\, |F_{ij}|$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

closed-loop performance   **controller sparsity**

$\gamma > 0$ – performance vs complexity tradeoff

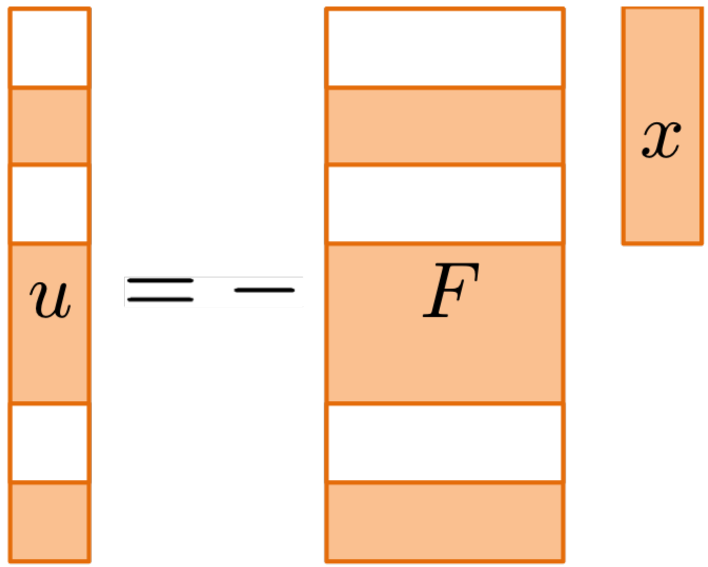
$w_{ij} \geq 0$ – weights (for additional flexibility)

Fardad, Lin, Jovanović, ACC '11

Lin, Fardad, Jovanović, IEEE TAC '13

Optimal actuator/sensor selection

- OBJECTIVE: **identify row-sparse feedback gain**

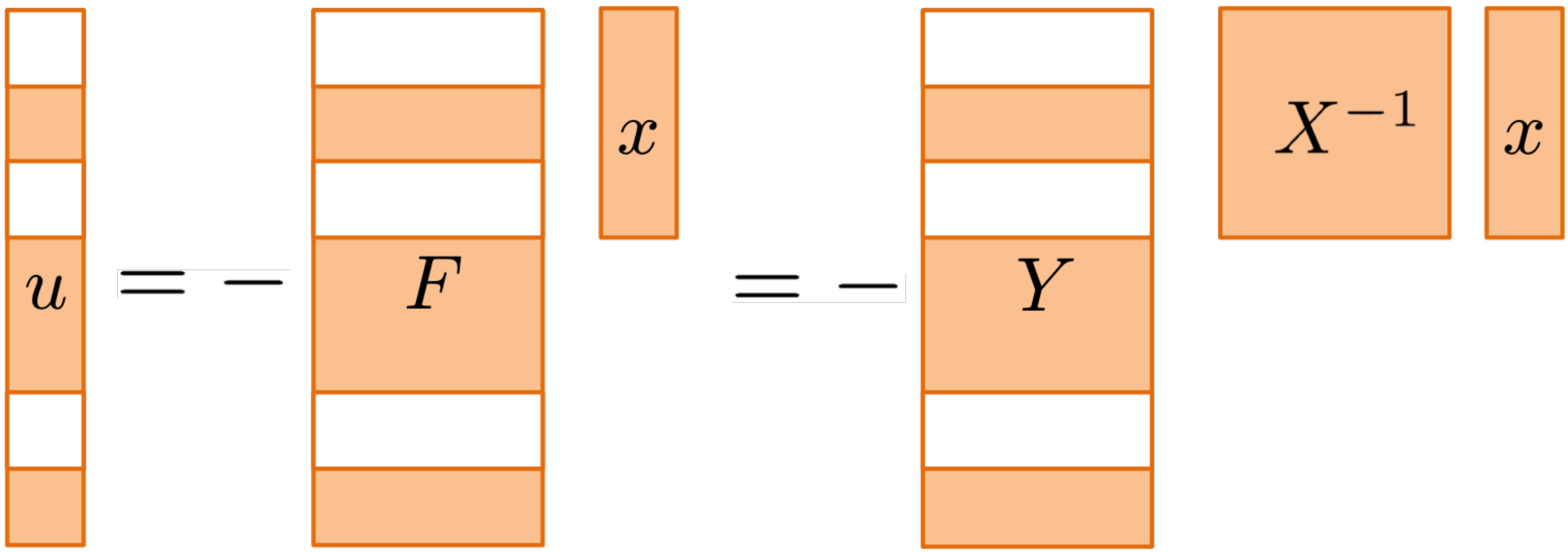


$$\text{minimize} \quad J(F) \quad + \quad \gamma \sum_i \|\mathrm{e}_i^T F\|_2$$

$J(F)$ ↓ **variance amplification**

$\gamma \sum_i \|\mathrm{e}_i^T F\|_2$ ↓ **row-sparsity-promoting penalty function**

- CHANGE OF VARIABLES: $Y := F\,X$
  - ⋆ **convex dependence of J on X and Y**
  - ⋆ **row-sparse structure preserved**



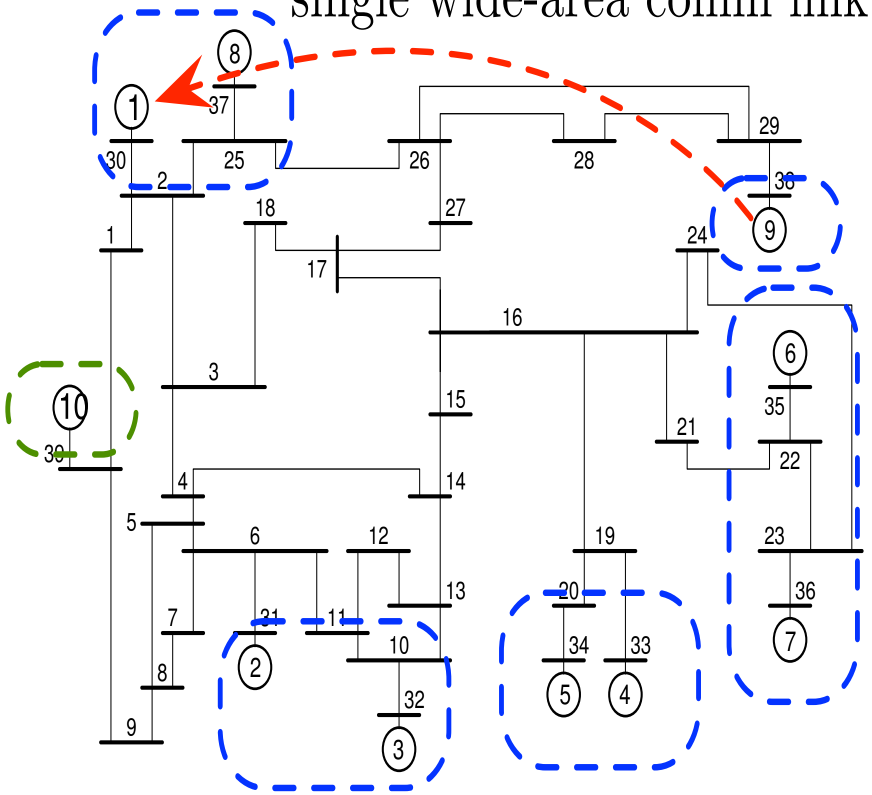
Polyak, Khlebnikov, Shcherbakov, ECC '13

Münz, Pfister, Wolfrum, IEEE TAC '14

Dhingra, Jovanović, Luo, CDC '14

Power networks

- SPARSITY-PROMOTING WIDE-AREA CONTROL
  - ⋆ **remedy against inter-area oscillations**



single long-range interaction:

nearly centralized performance

Dörfler, Jovanović, Chertkov, Bullo, ACC ’13

Dörfler, Jovanović, Chertkov, Bullo, IEEE TPWRS ’14

OPTIMIZATION

Augmented Lagrangian

Auxiliary variable

$$\begin{array}{ll} \underset{F,\,G}{\text{minimize}} & J(F) \;+\; \gamma\, g(G) \\ \text{subject to} & F \;-\; G \;=\; 0 \end{array}$$

⋆ benefit: decouples J and g

Augmented Lagrangian

$$\mathcal{L}_\rho(F,G;\Lambda) \;=\; J(F) \;+\; \gamma\, g(G) \;+\; \langle \Lambda, F \;-\; G \rangle \;+\; \frac{\rho}{2}\,\|F \;-\; G\|_F^2$$

Proximal augmented Lagrangian

$$\mathcal{L}_\rho(F, G; \Lambda) \;=\; J(F) + \underbrace{\gamma\, g(G) \;+\; \frac{\rho}{2}\,\|G \,-\, (F \,+\, (1/\rho)\Lambda)\|_F^2}\; - \;\frac{1}{2\rho}\,\|\Lambda\|_F^2$$

⋆ minimize over G

$$G^\star(F, \Lambda) \;=\; \mathbf{prox}_{(\gamma/\rho)g}(F \,+\, (1/\rho)\Lambda)$$

Proximal augmented Lagrangian

$$\mathcal{L}_\rho(F, G; \Lambda) \;=\; J(F) + \underbrace{\gamma\, g(G) \;+\; \frac{\rho}{2}\,\|G \,-\, (F \,+\, (1/\rho)\Lambda)\|_F^2}\; - \frac{1}{2\rho}\,\|\Lambda\|_F^2$$

⋆ minimize over G

$$G^\star(F, \Lambda) \;=\; \mathbf{prox}_{(\gamma/\rho)g}(F \,+\, (1/\rho)\Lambda)$$

⋆ evaluate $\mathcal{L}_\rho$ at $G^\star$

$$\begin{aligned} \mathcal{L}_\rho(F; \Lambda) \;&:=\; \mathcal{L}_\rho(F, G^\star(F, \Lambda); \Lambda) \\ &=\; J(F) \;+\; \gamma\, M_{(\gamma/\rho)g}(F \,+\, (1/\rho)\Lambda) \;-\; \frac{1}{2\rho}\,\|\Lambda\|_F^2 \end{aligned}$$

continuously differentiable

Method of multipliers

$$
\begin{aligned}
F^{k+1} &= \underset{F}{\operatorname{argmin}} \; \mathcal{L}_{\rho^k}(F; \Lambda^k) \\
\Lambda^{k+1} &= \Lambda^k \; + \; \rho^k(F^{k+1} \; - \; G^\star(F^{k+1}, \Lambda^k))
\end{aligned}
$$

- FEATURES
  - ⋆ nonconvex J: convergence to a local minimum
  - ⋆ F-minimization: differentiable problem
  - ⋆ adaptive ρ-update
  - ⋆ outstanding practical performance

Dhingra & Jovanović, ACC '16

Dhingra & Jovanović, arXiv:1610.04514

CONTROL-ORIENTED MODELING

Feedback flow control

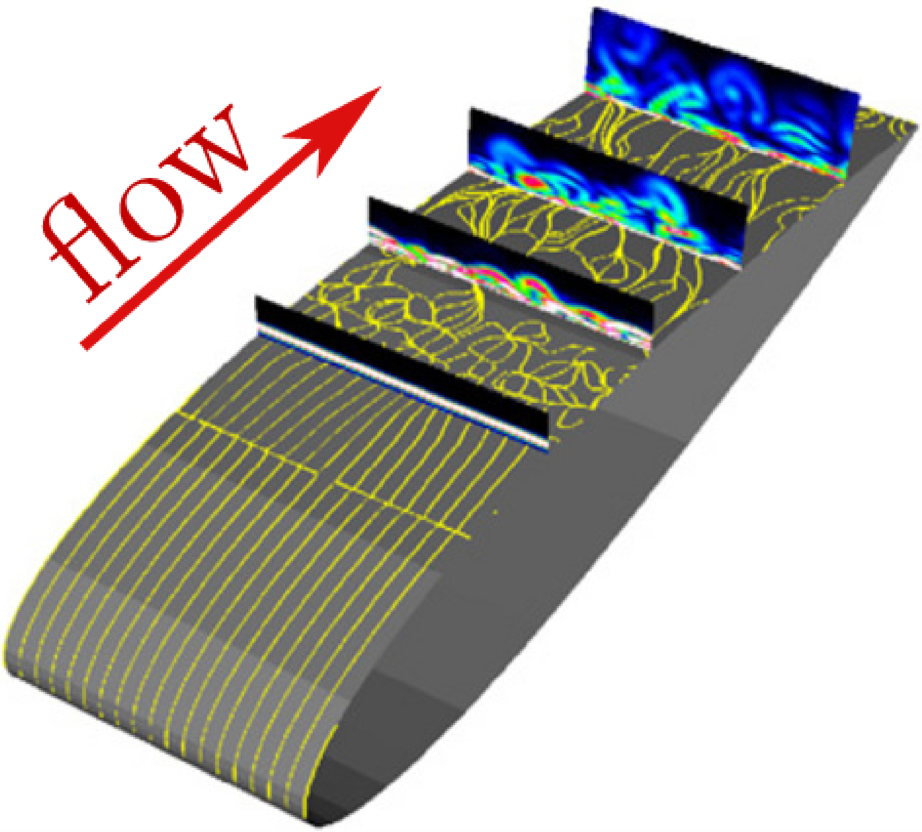


Image credit: M. Visbal

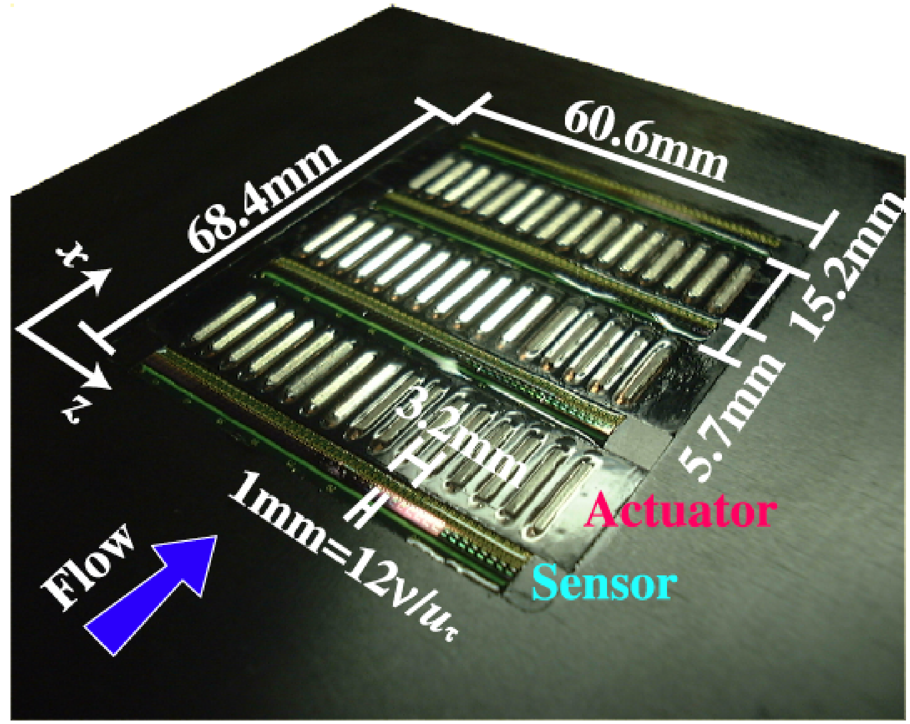


Image credit: Yoshino, Suzuki, Kasagi

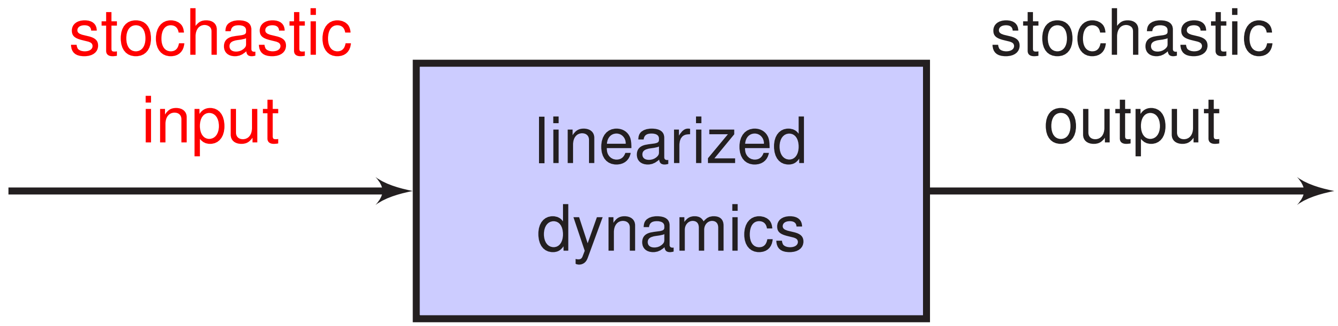
technology: **shear-stress sensors**; **surface-deformation actuators**

application: **turbulence suppression**; **skin-friction drag reduction**

challenge: distributed controller design for complex flow dynamics

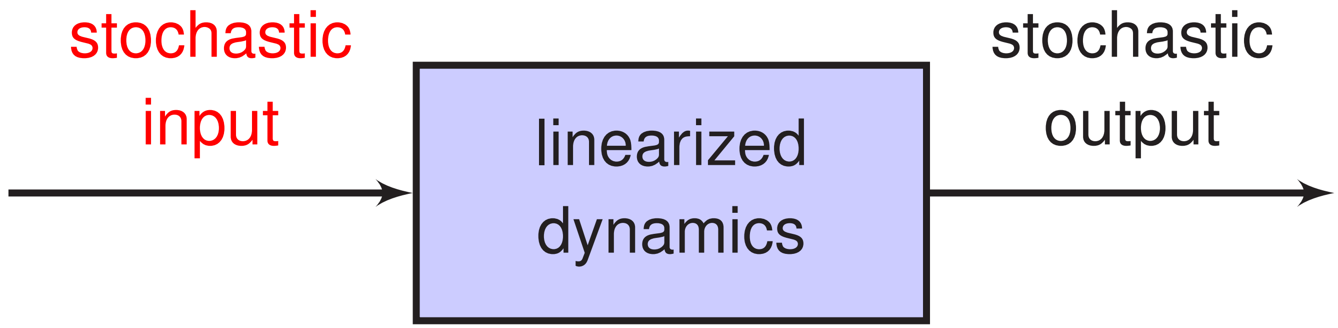
Low-complexity stochastic modeling

$$
\begin{aligned}
\dot{x} &= A\,x \;+\; B\,d \\
y &= C\,x
\end{aligned}
$$



Low-complexity stochastic modeling

$$\begin{aligned} \dot{x} &= A\,x \;+\; B\,d \\ y &= C\,x \end{aligned}$$



- **Objective**
  - ⋆ combine physics-based with data-driven modeling
  - ⋆ account for statistical signatures of turbulent flows using stochastically-forced linearized models

- THEOREM

$X = X^* \succeq 0$ is the steady-state covariance of $\dot{x} = A\,x + B\,d$

$$\Updownarrow$$

there is a solution H to

$$B\,H^* + H\,B^* = -(A\,X + X A^*)$$

$$\Updownarrow$$

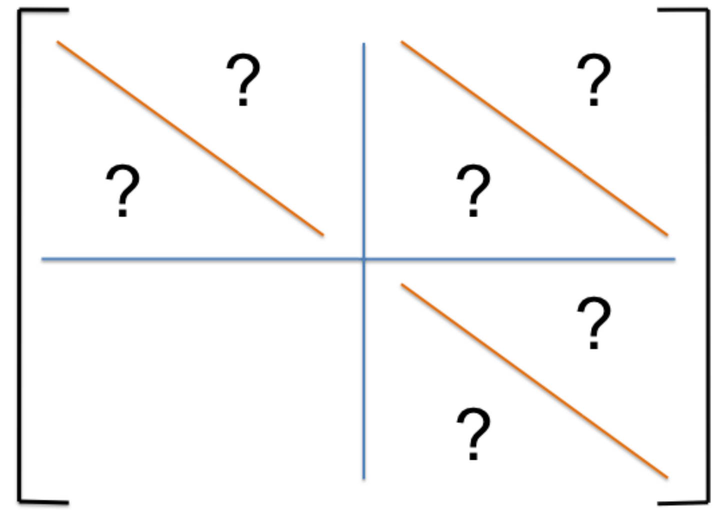
$$\text{rank}\begin{bmatrix} AX + X\,A^* & B \\ B^* & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} 0 & B \\ B^* & 0 \end{bmatrix}$$

Georgiou, IEEE TAC '02

Problem setup

$$A\,X + X\,A^* = -\underbrace{(B\,H^* + H\,B^*)}_{Z}$$

known elements of X



- **PROBLEM DATA**
  - ⋆ system matrix A
  - ⋆ partially available entries of X

- **UNKNOWNS**
  - ⋆ missing entries of X
  - ⋆ disturbance dynamics Z $\left\{ \begin{array}{l} \text{input matrix } B \\ \text{input power spectrum } H \end{array} \right.$

"Physics-aware" matrix completion

- CONVEX OPTIMIZATION PROBLEM

$$\begin{array}{ll} \underset{X,\,Z}{\text{minimize}} & -\log\det(X) + \gamma\,\|Z\|_* \\ \text{subject to} & A\,X + X\,A^* + Z = 0 \\ & X_{ij} = G_{ij} \quad \text{for given } i,j \end{array}$$

physics

available data

"Physics-aware" matrix completion

- CONVEX OPTIMIZATION PROBLEM

$$\begin{array}{ll} \underset{X,\,Z}{\text{minimize}} & -\log\det(X) + \gamma\,\|Z\|_* \\ \text{subject to} & A\,X + X\,A^* + Z = 0 \qquad \textbf{physics} \\ & X_{ij} = G_{ij} \quad \text{for given } i, j \qquad \textbf{available data} \end{array}$$

⋆ nuclear norm: proxy for rank minimization

$$\|Z\|_* := \sum \sigma_i(Z)$$

Zare, Chen, Jovanović, Georgiou, IEEE TAC '17

Zare, Jovanović, Georgiou, J. Fluid Mech. '17